UNIVERSITY OF TORONTO
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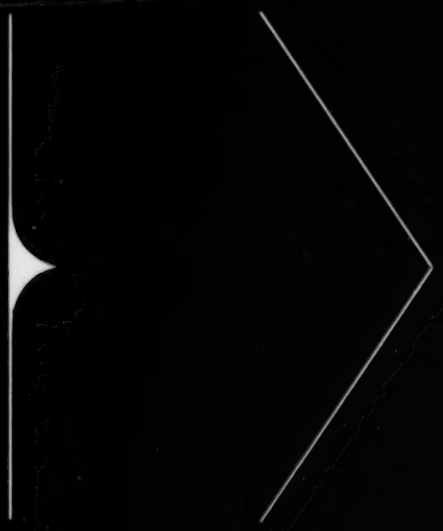
JOINT PROGRAM
IN TRANSPORTATION



PRICING AND INVESTMENT FOR TRANSPORTATION FACILITIES

RESEARCH REPORT NO. 80

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SEPTEMBER 1931

PRICING AND INVESTMENT FOR TRANSPORTATION FACILITIES

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Research Report No. 80

ptember 1981

ISSN 0316-9456

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ABSTRACT

Empirical models of an international airport and urban transportation system were developed to examine the effects of pricing and investment policies when passengers choose between substitute modes. Both models showed that there is a wide range of pricing and investment timing policies over which the economic welfare losses, in both absolute and percentage terms relative to the economically optimal policy, were not substantial. Policies leading to financial self-sufficiency for the airport would lie within this range; however, it is unlikely that such policies would be within this range for the urban transportation system, or that the public would accept the prices which would make it self-supporting. Peak load pricing will slow the rate at which capacity is added, particularly on the urban transportation system, where it will induce passengers to substitute public transit for the automobile.

EXECUTIVE SUMMARY

The objective of this report is to investigate the effects of various pricing and investment timing policies on economic welfare, financial performance, and capacity utilization for transportation facility networks.

Based on a review of the literature, it was decided that the best way to approach this problem was not by building a large transportation network model, but by focusing on a smaller model that would deal with these issues for transportation facilities which are substitutes for one another. The literature on the economics of the second-best has been applied to a version of this problem, namely the optimal pricing policy for public transportation when private transportation is not charged marginal cost because of the absence of congestion pricing for roads. While a theoretical literature has developed to deal with this problem, the empirical literature has not kept pace.

In section 3 of this report, a theoretical model to investigate these problems was presented. It drew on the recently developed translogarithmic demand model to formulate the inter-related demand functions which would characterize a network of substitutable transportation facilities. Two empirical applications of the model were developed, an international airport, and an urban transportation system consisting of an expressway and a subway which are substitutes for one another. The models were based on data for airports, expressways, and subways in Toronto. The airport model dealt with substitution between peak and off-peak flights, the urban transportation model with substitution between expressway and subway trips at both peak and off-peak periods.

The major results derived from the airport application were:

1. Losses due to non-optimal pricing and investment were small in relative terms (generally less than 1 per cent of the total net present value of economic surplus for optimal policies), but more substantial in absolute terms. For example, the existing pricing policy results in a welfare loss of $50 million over the model's 40-year planning horizon, or approximately $5 million per annum. The higher the assumed demand elasticity, the greater the relative welfare losses; however, absolute losses are not affected by the demand elasticity. Substantial deviations in pricing and investment policies, such as using an interest rate of .5% or 50% to determine when to add capacity or charging aircraft passenger and landing fees of $5,000 per flight, result in much larger absolute welfare losses, closer to $500 million over 40 years.

2. It is relatively easy for the airport to be financially self-supporting. This would require approximately a doubling of the current landing and passenger fees of $1,000 per flight, which would increase the welfare loss from $50 million to $100 million over 40 years.

The major results derived from the urban transportation application of the model were:

1. Losses due to non-marginal cost pricing and non-optimal investment timing policies were relatively very small, and absolutely quite small as well. For example, the loss due to existing pricing policies rather than marginal cost pricing of the subway and expressway is about $15 to $20 million over 40 years. Relative welfare losses increase with the

assumed elasticity of the demand function: absolute losses do not.

2. Pricing policies that treat the subway and expressway equally (such as charging the same fee for either, whether it is 60¢ per trip or nothing) do better than the current policy of charging 60¢ to use the subway while charging nothing on the expressway. Also, peak period pricing policies, such as charging 60¢ on both the subway and expressway during the peak period and nothing during the off-peak, do better than either existing policies or any policy that charges the same price throughout the entire day.

3. It is quite difficult for the urban transportation system to be financially self-sustaining. A fee of $2 to $3 per trip, on either facility and at all times of day, would achieve self-sufficiency, with profits on the expressway exactly covering losses on the subway. While this would increase the absolute welfare loss to $100 million over 40 years, it is most unlikely that such a policy would be politically acceptable.

4. The model shows that the balance of traffic between the expressway and the subway is quite sensitive to the relative prices charged. Marginal cost pricing policies or peak-period pricing policies shift a substantial amount of traffic from the expressway peak to the subway, with the result that there is less need to expand expressway capacity.

The overall result that characterizes both these models can be stated metaphorically. The economic welfare surface that surrounds optimal pricing and investment policies for both models is relatively flat for a certain range. That is to say, deviations from optimal pricing and investment policies will be quite small in relative terms and acceptable in

absolute terms. Assuming that the real interest rate (net of inflation) which is socially optimal is 10%, using rates between 5 and 15% for capacity expansion decisions will have little effect on economic welfare. In terms of pricing, anything up to about $2,000 per aircraft flight (which is double the existing level) or $1.50 to $2.00 per subway and expressway trip will have little effect on economic welfare. It is possible for the airport to be financially self-sufficient at prices within this range. The urban transportation system could be financially self-sufficient at prices at, or perhaps a bit beyond, the upper limit of that range. However, such prices are probably not politically acceptable. It is likely that urban transportation systems (including subways and expressways) will continue to require subsidies from general tax revenues.

If a transportation authority is concerned about keeping its capital budget under control, the best way it could do so would be to charge more during peak than off-peak periods. The urban transportation model shows that such differentiated charges reduce demand for the expressway during the peak, and therefore reduce the rate at which the expressway must be expanded. On the other hand, the existence of excess capacity on the subway system means that this can be done without having to expand the subway.

Transportation policy makers, therefore, are urged seriously to consider peak-period charges as a way of improving the utilization of capacity and reducing capital budget needs.

ACKNOWLEDGMENTS

A number of individuals and organizations were instrumental in the production of this report. The funding was provided by Transport Canada, under its contract research program. Administrative arrangements were handled most capably by the University of Toronto - York University Joint Program in Transportation. Nancy Stenson took care of the financial management of the project, and Ann Poole assisted in getting a literature search underway. Margot Danard helped with the search and review of the literature. The computing work was done on the Faculty of Administrative Studies' VAX computer: the staff at the FAS Computer Centre, particularly Robert Gagne, were most helpful. The final version of the report was most capably typed by Linda Clemow. Finally, David Gillies did a superb job in programming and running the models upon which this report is based. Without his assistance, this report truly would not have been possible. Of course, I assume full responsibility for the opinions expressed here, as well as any errors that remain.

CHAPTER ONE
INTRODUCTION

1.1 Previous Research

This research is an extension of my previous research on pricing and investment for transportation facilities, which was presented to Transport Canada in Borins (1980) and is to be published for academic audiences in Borins (1981), which is attached as Appendix A.

My previous research examined the consequences of non-optimal pricing and investment policies for simple models of transportation facilities. The major simplification was that the models involved a single facility, or bottleneck, and thereby avoided the complexities of full transportation networks. Two models were developed, using parameters that were felt to be representative of a major international airport and an urban expressway.

The following were the major results of the previous study:

1. In both the models, the losses due to non-marginal cost pricing are a small percentage, almost always less than 1%, of the value of economic surplus for marginal cost pricing. In the airport model, the absolute losses due to the existing pricing policy are fairly substantial (between $10 and $40 million in net present value over forty years) while in the highway model they are negligible (about $150,000 in net present value over forty years). The more elastic the demand function, the greater the relative and absolute losses. The only instances where substantial relative losses were incurred were when the demand function was elastic and when very large user charges (airport landing fees and taxes of $4,000 to $10,000 as compared to the

existing level of $1,100) were levied.

2. In both models, the relative losses due to the non-optimal timing of investments were also small. In this case, optimal capacity expansion occurs if an interest rate equal to the "social opportunity cost" of capital of 10% is used to determine when to expand capacity. Non-optimal interest rates as low as .5% and as high as 50% reduced economic surplus by less than 1% relative to that which resulted for the correct interest rate of 10%. Relative losses were somewhat higher for more elastic than for less elastic demand functions. It was also found that the losses due to non-optimal pricing and non-optimal investment timing were additive.

3. Marginal cost pricing policies do not recover the full capital cost of the facilities, but rather cover the capital cost of all expansions subsequent to the initial construction of the facilities. However, it was found that the facilities could be made self-sufficient by using the same type of user charges as presently in effect, but with higher rates. In the airport case, this would mean approximately doubling the present level of user charges. The welfare losses due to user fees of this magnitude were not substantial.

An important question that arises is whether several transportation facilities should be financed separately or collectively. For example, the model suggests that the new Mirabel Airport will probably never be self-sufficient because present user fees will not pay for the initial facility, even though they may pay for all subsequent expansions. However, the Airports Revolving Fund, which handles financing for Canada's major airports, was set up without any capital charges for the initial facilities. If revenues

and costs from the older airports (Toronto, Dorval, and Vancouver) were pooled with those from Mirabel, the latter would be considered as an increment to the entire system, which could then come close to self-sufficiency.

4. The effects of various pricing policies on the timing of capital expansions were also examined. When high interest rates are used in investment decisions, the expansion dates can be delayed substantially relative to low interest rates. For example, an interest rate of .5% advances the opening date of expansions to the airport by approximately fifteen years relative to an interest rate of 50%. However, the difference between interest rates of 5 and 15%, which bracket the range policy makers are likely to use, is three to five years.

5. In the expressway model, a number of different congestion cost functions were used. It was found that those which assume substantial congestion costs long before capacity of existing facilities is reached lead to much more expansion than those where congestion costs do not become significant until existing facilities are close to capacity. This underlines the importance of accurate estimates of congestion costs, as based on volume-delay curves.

6. Even though peak and off-peak prices were assumed to be mutually independent, the model was used for both peak and off-peak periods in order to see the shift in capacity between peak and off-peak periods as a function of the various pricing policies. It was found that marginal cost pricing resulted in a more even balance of traffic between peak and off-peak periods than did existing pricing policies, which charged the same fee to all users during all years of the life of the facility.

The most dramatic overall conclusion can be explained metaphorically. These models suggest that the economic welfare surface around the optimal pricing and investment policies is rather flat. Thus, a substantial range of deviations from marginal cost pricing and investment timing using the socially optimal discount rate will not result in very great relative welfare losses. This implies that transportation pricing and investment policies can be used to achieve other economic policy objectives with little loss in efficiency. The example discussed in the previous report was financial self-sufficiency: projects could break even, or even become profitable, with little loss in welfare.

1.2 Present Research

The objective of the present project is to determine whether these results can be generalized to more complicated transportation systems. Transportation networks involve two complications relative to simple facilities. First, the user travels through many different facilities and bottlenecks between his origin and his ultimate destination. Second, users are presented with many choices as to how to travel between origin and destination. The choices can refer to choices between modes (for example, air or rail in inter-urban travel, and expressway or public transit in intra-urban travel), between routes (two different sets of roads to travel between home and work), or between periods of time (peak and off-peak).

In the following sections of this report, the literature on the modelling of these more complicated transportation networks is discussed, an expanded model of a more complicated transportation system is outlined and then specified for a major airport and an urban transportation

system, and the effects of non-optimal pricing and investment policies are examined. Thus, the report attempts to determine whether the same general conclusions of the previous report hold when a more realistic transportation model is used.

CHAPTER TWO
LITERATURE REVIEW

2.1 Transportation Network Models

The literature on transportation networks indicates that transportation network models can become very complicated computationally, but that their economic content is not substantially greater than that of simple models. An example of a complicated computational network is LeBlanc, Morlok, and Pierskalla (1974) who developed an empirical model of the road network in Sioux Falls, South Dakota, a city of approximately 125,000 residents. The network, consisting of 76 arcs and 24 nodes, requires 6 seconds of CPU time on a CDC 6400 computer to solve by means of a non-linear programming algorithm. It was felt that the time and expense which would be required to use this model, or to develop one like it, would be prohibitive given the size of the grant supporting this research.

The economic structure of transportation network models was first discussed on a theoretical level by Littlechild (1974), who reached the following conclusions:

> the equilibrium condition of the model can be easily stated. For each origin and destination pair, the marginal value of a trip is equal to the sum of the marginal costs on the cheapest route or routes; routes with higher costs are not used. For a 'controlled' network the marginal costs referred to are social costs; for a 'user optimised' network they are private costs. What is immediately apparent from the formulation is that a user-optimised network can be led to optimality by a toll imposed on each link, calculated in the way described earlier for a single road. The total toll paid by any traveller therefore consists of the sum of the tolls on the route he chooses. For each route used by travellers between a specific origin and destination pair,

> the total cost including time and tolls will be the same, and less than on any route not used by them. (Littlechild, 1974, p. 201)

Smith (1979) reached essentially the same result, except that he generalized it somewhat, namely to show that it would hold so long as the demand functions in the model are continuous and the cost functions have continuous partial derivatives.

Considering my previous research in this light, it became clear that, in a model of a transportation network, users would simply be moving through a series of bottlenecks, rather than one. If the deviations due to non-marginal cost pricing on one bottleneck are relatively small, then the deviations experienced on a series of bottlenecks would also be relatively small. Thus, given the difficulty of developing a network model that would be very realistic, and the theoretically predicted similarity between the results of such a model and the results of the simple model, I decided not to pursue this line of research any farther. In addition, as will be shown in the previous section, theoretically more interesting problems have been posed in more limited models of competing transportation modes.

2.2 Models of Competing Transportation Facilities

Economists have become interested in the pricing of competing transportation facilities because this problem is one illustration of an idea which has attracted a great deal of attention in the last two decades, namely the economics of the 'second best'. The general theorem of the second best states that, if there are a number of distortions from economic optimality in an economy, correcting any subset of

them may not make the entire economy better off. In order to decide the best course of action, a full constrained maximization of economic welfare must be undertaken.

Initially, second-best models examined instances where a number of goods had to be taxed in order to raise a fixed sum of revenue, presumably to pay for pure public goods provided by the government. The taxes created distortions between the prices consumers paid for the goods and their actual marginal cost of production. The second-best models showed that the optimal second-best tax pattern depended on the elasticities and cross-elasticities of demand among the various goods. If cross-elasticities are assumed equal to zero, then the ratio of price to marginal cost for all goods should be equalized. When cross-elasticities are allowed to deviate from zero, results become more complicated. One key result, however, is that substitute goods should be treated equally: that is, if one of a pair of substitutes must be taxed, the other should also be, so as to create the same relative deviation between price and marginal cost. On the other hand, if a pair of goods are complements, if one is taxed, the other should be subsidized, so as to bring price as near as possible to marginal cost for the pair.

Transportation economists began to apply the theory of the second best to an important policy problem in urban transportation. We know that urban roads are not rationed by means of marginal social cost pricing, but that travellers simply incur average social cost. The implication of this (see Borins, 1980) is that, when roads are congested, motorists are paying less than marginal social cost and, assuming any elasticity in the demand curve, are using the roads to a greater degree than is socially optimal. Public

transportation is a substitute for the use of private automobiles on the roads. This is true whether the public transit uses dedicated rights-of-way (such as subways or buses using dedicated lanes) or whether it shares the road with automobiles (such as buses). Roughly speaking, the general theorem of the second best would lead us to conclude that, if drivers of private automobiles are not paying their full marginal cost, then passengers on public transit should also not be required to pay full marginal cost. This serves as a possible justification for the subsidization of public transit. Since the roads are not required to be financially self-sufficient, why should a public transit system be financially self-sufficient? Thus, according to this line of reasoning, critics of public transit who call for higher fares to achieve financial self-sufficiency are mistaken, and their policies will only diminish economic welfare.

Initially, this problem was first examined by economists on a theoretical level, so as to determine the precise conditions for subsidization of public transport. Papers along these lines were written by Marchand (1968), Sherman (1971 and 1972), Glaister (1974), and Bertrand (1977).

Marchand (1968) considered a case where two roads are perfect substitutes but one (a tollway) is taxed while the other (a freeway) is not. His conclusion is that "the optimal toll must be such that the reduction of misallocation on the tollway, when the toll is raised an infinitesimal amount, exactly offsets the corresponding increase in misallocation on the freeway" (Marchand, page 579). Also, because the freeway is not taxed, the optimal toll on the

tollway would be less than required to equate price and social marginal cost on the tollway.

Sherman (1971) extended Marchand's analysis by incorporating separate demand functions and, hence, the possibility of imperfect substitution between modes, and also congestion interdependence. The latter would occur when, for example, both buses and automobiles share the same roadways. The model specifies that automobile passengers make a greater contribution to congestion than bus passengers because automobiles almost always carry fewer passengers than buses.

Sherman first considers a case where there are no input taxes (e.g., gasoline taxes) on either mode and where automobile drivers bear only average cost. In this case, "the price of public transit should be set lower as the cost elasticities are relatively higher for auto passenger miles. And as a decrease in the price of public transit elicits a greater response from auto travelers the price should be set lower, even to the point of requiring a subsidy, for a lower fare can then reduce overall congestion." (Sherman, 1971, p. 572). If the two modes are perfect substitutes, then the bus should definitely be subsidized, that is, the price should be less than average cost. Sherman then adds the possibility of an inputs tax on both bus and auto. He finds that, "with an optimal inputs tax, a lower price is optimal for public transit, even to the point of causing a loss, as the expenditure for auto travel [becomes] relatively smaller" (1971, p. 573). However, the loss for public transit can be offset by the proceeds of the input tax. If the inputs tax is effective both at peak and off-peak hours (there being no congestion during the latter), then "the

off-peak public transit price should be lower, even to the point of causing a loss, as the expenditure for auto travel [becomes] relatively smaller" (1971, p. 574). As an example of Sherman's qualitative results, he finds that, if there is no inputs tax, then the "second best rush-hour public transit price will lie below average cost and require a subsidy if the ratio of cost elasticities with respect to bus and automobile trips is smaller than the ratio of (absolute values of) compensated demand elasticities for automobile and bus trips in response to changes in the price of bus trips" (1971, p. 575).

In a subsequent paper, Sherman (1972) looked at actual estimates of the cost and demand elasticities so as to apply his qualitative results to the real world. He concludes "what data exist seem to suggest that a subsidy to buses would be appropriate in London, to be financed out of taxes on transit inputs. The same may be true for large U.S. cities. Results of this analysis are less applicable there, however, because, although taxes exist, they are not at the level assumed, which would equate the private and social marginal costs of peak travel by private car." (1971, p. 30).

Glaister (1974) generalized Sherman's model to include four modes, public and private transit in both peak and off-peak periods. In his model, the only policy variables available to the decision-maker to maximize social welfare are the peak and off-peak public transit fares. He concludes that the optimal prices for public transit should be below marginal social cost in both the peak and the off-peak periods (1974, p. 859). However, it is not necessarily the case that peak period charges for public transit should be higher than off-peak period charges. Like Sherman's

paper, the exact results depend on demand elasticities, cross-elasticities, and the elasticity of total social cost with respect to the different modes of transportation.

Bertrand (1977) generalized the previous results by presenting a general equilibrium model in which there is a non-transportation sector as well as several transportation modes. He assumed that there were no distortions in the non-transportation sectors, but there were distortions in the transportation sector (such as untaxed congestion on one mode). If transportation taxes do not affect the total number of passengers on all modes combined, but do affect the distribution of passengers among modes, then second-best congestion taxes require that the distortion between the congestion fees and marginal costs imposed on others per passenger mile be equated across all modes. (1977, p. 1708). This formulation, like the others, provides a rationalization for subsidizing public transit if automobile congestion is untaxed. If raising any congestion tax on a particular mode reduces aggregate transportation, then congestion taxes on all vehicles not subject to the constraint that no taxes can be charged should be greater than or equal to the rate that equalizes distortions per passenger mile across modes. The latter result can be intuitively explained because higher fees will reduce the demand for total transport services for which the benefits fall short of the costs at the margin.

Bertrand's model carried the theoretical work about as far as possible. The next logical step in this research is the estimation of empirical models that would determine how much public transit should be subsidized, how great a deficit would result, and whether the set of prices dictated by

these second-best models differs substantially from existing prices and would lead to substantial welfare gains relative to current pricing policies. Some of these questions were explored in papers by Jackson (1975), Glaister and Lewis (1978), and Bertrand (1978).

Jackson developed a simple model in which there are two substitute modes, highways and public transit. Highways are untaxed, so that travellers pay average social cost, rather than marginal social cost. Public transit is not subject to congestion, so that average and marginal cost are identical (a horizontal line). This assumes that there is congestion independence and that the public transit system has substantial excess capacity. If the price of public transit is lowered below average (or marginal) cost, some passengers will be attracted from the highway, thereby reducing the externality in the highway sector. However, a new externality is created in the public transit sector because price is now lower than marginal social cost. The objective of the government's public transport pricing policy is to reduce the aggregate externality. Jackson then computes the optimal subsidies for public transit, given various assumptions about the cross-elasticity of demand between the highway and public transit and the extent of the divergence between marginal social and average cost on the highway. For small divergences regardless of the cross-elasticity second-best pricing does little to improve economic welfare. For example, when marginal cost is 20% greater than average social cost, the optimal subsidy is only 2% of the public transit fare, and the welfare loss given second-best pricing is still 99% of the welfare loss without it. Only when there is a high cross-elasticity of demand <u>and</u> a substantial divergence between marginal and average social cost, does

second-best pricing have very much impact. The largest impact Jackson found was when the cross-elasticity of demand was .6 and marginal social cost was 120% higher than average social cost. In this case, the subsidy would be equal to over 40% of the original public transit fare, and the externality with second-best pricing was 71% of the previous externality (1975, p. 9).

Glaister and Lewis (1978) built a more ambitious model, estimating second-best prices for automobile, bus, and rail transportation in both peak and off-peak periods for the Greater London area. The model's estimates of own- and cross- elasticities were based on econometric studies and several estimates of marginal social costs. The authors used both short-run and long-run marginal social cost estimates for bus and rail. The long-run estimates were higher because of the need for additional capital in the future. The short-run second-best model results in somewhat lower bus fares than at present, and substantially reduced rail fares. There is a great increase in rail traffic, some decline in bus and automobile traffic, and a total annual subsidy required of about 100 million pounds. In the model which uses long-run bus and rail costs, bus and rail fares are substantially higher, though still not equal to marginal social cost (because automobile fares are also not equal to marginal social cost). Auto traffic increases slightly, bus travel stays constant, and rail travel is reduced during the peak but increased by an equal amount in the off-peak. The total annual subsidy would rise to about 200 million pounds. In their conclusion, the authors point out that most likely estimate of the policy's impact would not lead to a subsidy much greater than 150 million pounds per annum, and that their policy would result in lower rail fares relative to

bus fares, in order to encourage a redistribution of traffic from bus to rail, since the latter is less expensive. The authors conclude that they would be interested in comparing these results with those of some sort of "first best" policy, such as road pricing; their expectation is that the second-best policies would indeed be inferior in terms of efficiency to the first best policy.

Bertrand (1978) applied his theoretical model to the transportation system in Bangkok. He looked at seven different types of vehicles using the streets of Bangkok: buses, cars, taxis, motorcycles, samlors (small three-wheeled taxis), light trucks, and heavy trucks. He constructed a general equilibrium model of the type described in his paper using estimates of key empirical relationships, such as cost characteristics of various vehicles and volume-delay curves. He found that there was substantial congestion and that costs imposed on others by all vehicles were very high, substantially higher than the taxes actually paid. Bertrand then estimated the effects of moving to an optimal taxation system. The overall vehicle use would decline from 68 to 60 million vehicle-kilometres per week. Car travel would decrease 7 percent, while bus travel could increase 6 percent, and taxi trips by 40 percent. The net welfare gain would be $160,000 per week (U.S.). Bertrand broke the welfare gain down into the tax revenues from the new taxation system and loss in economic surplus due to the reduction in travel. There would be a loss of 60 million baht (the Thai currency, equal to $.05 U.S.) per week in consumer surplus because of trips no longer taken, while tax revenues would increase by 63 million baht per week. Bertrand doubts that the proposed system would be implemented unless the government were to find some way of returning the increased tax

revenues to the travellers, who will now be travelling less. Bertrand also calculated second-best solutions where the congestion tax on private cars was constrained to 75 and 50 percent, respectively, of the external costs of private car use. The former resulted in a welfare gain of $151,000 U.S. per week, which is almost equal to the optimal taxation system. The latter resulted in a welfare gain of $113,000 U.S. per week. Bertrand finds that in all cases buses would be taxed more heavily than at present. This result, which seems to contradict the previous research, is explained because the entire system is congested, and there is assumed to be a very low cross-elasticity of substitution between buses and automobiles. Bertrand compared the welfare gains to the total cost of the transportation system, and finds that, for his three solutions, they vary between 1.5% and 2.3% of total cost, which leads him to conclude

> the welfare gain is thus equivalent to a one-shot improvement in transport activities which reduces costs by less than 2½ percent, and if the tighter constraint on car taxation is effective, as it is likely to be in Bangkok, by about 1½ percent. Viewed in this perspective, it again appears probable that the costs of implementation or slippages from the types of taxes discussed here will be enough to eliminate net gains (1978, p. 362).

However, Bertrand ends on a slightly more optimistic note by suggesting that, while his model applies to the entire urban transportation system, taxes might be more feasible or effective at certain key bottlenecks in the system. Furthermore, because Thailand is a developing country, wages and hence the value of passengers' time is low. In wealthier countries, time savings could have a greater value, which might lead to more acceptance of such pricing schemes.

Are there any generalizations which emerge from the three models examined thus far? Both Jackson and Bertrand indicate that the benefits of introducing optimal or second-best pricing schemes, at any point in time, are not large. Jackson found that second-best pricing did not reduce existing externalities very much, and Bertrand found that both optimal and second-best pricing did not result in great efficiency gains relative to the total transport cost. These findings are consistent with my previous research, and should lead us to look carefully for any situations where a more sophisticated model could lead to a more significant impact for optimal or second-best pricing policies.

In all three cases, the models are static, in that they look at the benefits of better pricing policies at a given point in time, and do not consider the dynamic question of the effects of various pricing policies on investment decisions, and their cumulative welfare effects over time. Furthermore, the difficulty of developing the models (especially Glaister and Lewis (1978) and Bertrand (1978)) left little scope for experimenting with a wide variety of pricing policies. The next section will develop a model which will continue the emphasis on examining relative welfare gains due to better pricing policies, but which will attempt to do so in a dynamic context, which relates pricing and investment decisions.

CHAPTER THREE
THE MODEL

3.1 Demand Function

Since the model deals with the demand for inter-related transportation modes, the prices of the competing modes will enter into each others' demand functions. The previously discussed empirical papers have evaluated the incremental benefits of moving from one pricing policy to another as the line integral of a demand function where the cross-substitution effects are equal to one another. However, these models have not attempted to calculate a "baseline" level of consumer surplus to which the increments due to changes in pricing policy or capacity expansion could be compared.

In order to do this, I have used a recently developed demand function, the transcendental logarithmic (or trans-log) demand function. For an exposition of the trans-log utility function, see the original article presenting it, Christensen, Jorgenson, and Lau, 1975. The trans-log model has been employed to explore the nature of transportation demand functions for travel in Canada in papers by Oum and Gillen (1980A, 1980B). The application of the trans-log function in this study is based on Hogan (1980), who used it to study the impact on regulation of exhaust emissions in the United States.

We begin by assuming a simple logarithmic demand function for travel of the form

$$Q = a_0 P^{-b} \qquad (1)$$

where Q is the quantity of trips taken, P is the price, and a_0 and b are parameters of the demand function.

The wrinkle in this model is that Q is an aggregate of the various types of trips taken, and P represents their overall cost. P can thus be thought of as a weighted average of the prices of the various substitute modes employed in the model.

The trans-log model makes this aggregation more specific. We can write a translog cost function of the form

$$\ln P = A + \sum_i B_i \ln P_i + \tfrac{1}{2} \sum_i \sum_j C_{ij} \ln P_i \ln P_j \qquad (2)$$

where P is as defined in equation 1, P_i is the price of transportation by mode i, and A and B are parameters. The trans-log cost function allows for substitution by users among the various modes. For simplicity, we would write a two mode model as

$$\ln P = A + B_1 \ln P_1 + B_2 \ln P_2 + \tfrac{1}{2} C_{11} (\ln P_1)^2 + \tfrac{1}{2} C_{12} \ln P_1 \ln P_2 + \tfrac{1}{2} C_{22} (\ln P_2)^2 \qquad (3)$$

The competitive share values for each mode (S_1 and S_2) are simply the logarithmic derivatives of the cost function:

$$S_i = \partial \ln P / \partial \ln P_i = B_i + \sum_j C_{ij} \ln P_j \qquad (4)$$

In the two mode model,

$$S_1 = \partial \ln P / \partial \ln P_1 = B_1 + C_{11} \ln P_1 + C_{12} \ln P_2 \qquad (5)$$

$$S_2 = \partial \ln P / \partial \ln P_2 = B_2 + C_{12} \ln P_1 + C_{22} \ln P_2 \qquad (6)$$

Since $S_i = Q_i . P_i / Q . P$, we can write

$$Q_i = (S_i . P . Q) / P_i \qquad (7)$$

which gives us a trans-log demand function for trips by mode i. Thus we have a set of i demand functions for the i modes which can now be solved simultaneously. Before doing so, let us consider the nature of the congestion cost functions.

3.2 Cost Functions

The prices faced by the passengers using the different transportation modes are defined as follows:

$$P_i = (F_i + t_i + Z_i(C_i/(C_i-Q_i))^x) \text{ where } C_i > Q_i \qquad (8)$$

where P_i is the cost to the average user of travelling by mode i, F_i is a constant, which represents the real resource costs of the trip which are not a function of congestion on mode i, t_i is any toll or user fee the government may charge on mode i, Z_i is constant, equal to the time and operating cost to the average user of using mode i where it is completely uncongested, C_i is the capacity of mode i, and x is a parameter of the congestion cost function.

When the mode is completely uncongested, Q_i is very small and the expression $C_i/(C_i-Q_i)$ is approximately equal to 1. As the mode becomes more crowded, the value of $C_i/(C_i-Q_i)$ increases, thus raising the average cost of using the mode. The condition $C_i > Q_i$ allows all users to be served at mode i by the end of the period. For the case where $x = 1$, the average cost function is that which is derived from steady-state queuing theory (De Salvo and Lave, 1968). For values of x smaller than 1, congestion costs do not increase very much until the mode is operating close to capacity. It has been found that airport runway congestion closely approximates the steady-state queuing model where

$x = 1$, while expressway congestion cost functions are better approximated by using a low value of x, such as .2.

The cost functions can also be used to show the difference between marginal cost and various other pricing policies.

The total cost of all trips made by mode i is equal to

$$Q_i(F_i + Z_i(C_i/(C_i-Q_i))^X) \tag{9}$$

This excludes t_i, since it is a user fee, rather than a real resource cost.

Taking its derivative with respect to Q_i yields

$$F_i + Z_i(C_i/(C_i-Q_i))^X.(1 + x.Q_i/(C_i-Q_i)) \tag{10}$$

which can be reduced to

$$F_i + Z_i(C_i/(C_i-Q_i)^2 \tag{11}$$

when $x = 1$.

In order to implement marginal cost pricing, it is necessary to charge a toll equal to the difference between marginal and average cost, which is

$$Z_i(C_i/(C_i-Q_i))^X(1 + x.Q_i/(C_i-Q_i)) -$$

$$Z_i(C_i/(C_i-Q_i))^X \tag{12}$$

which equals

$$Z_i(C_i/(C_i-Q_i))^X(x.Q_i/(C_i-Q_i)) \tag{13}$$

which is 0 when Q_i is zero, and increases as Q_i increases.

3.3 Solving the Model

In order to solve the model, we would insert the equations representing the P_i, equation 8, into the trans-log demand functions (7). We would also substitute for P, which is a function of the P_i. Thus, we would have i demand equations, all in terms of the Q_i, since in each case we have substituted for P_i an expression in terms of Q_i. Given the interdependent demand functions, all the different Q_i's will be present in each demand function, so that they all must be solved simultaneously. Thus we have i simultaneous non-linear equations in i unknowns. Determining the equilibrium quantities of the various Q_i's enables us to determine the equilibrium prices. Finally, once we have an equilibrium value of P, which is the "overall" price of trips, we can plug it back into the original demand function (1) for trips, and solve that function for Q.

The P that emerges will be a weighted average of the P_i, with each weighted (approximately) by its corresponding Q_i. The Q that results is roughly equal to the sum of the Q_i.

The simultaneous equation system was solved by means of Newtonian approximation. The solution procedure went roughly as follows. An initial guess was made at the values of the quantity vector Q. Then a Jacobian matrix of the form $J_{ij} = \partial f_i / \partial q_j$ was formed where f_i equals the trans-log demand function i minus P_i. Taylor's theorem, expressed in n-dimensions, states that the next guess at the solution should be

$$Q^n = Q^o - J^{-1}f \qquad (14)$$

where Q^n is the new estimate of Q and Q^o the previous estimate.

Due to the difficulty of inverting large matrices, we set the matrix $Y = J^{-1}f$. By linear algebra, it was solved for Y. Thus, the next guess was $Q^n = Q^o - Y$. When the model converged on equilibrium solutions of Q, it converged quadratically. It did not converge for certain values of the parameters, as will be discussed below.

Appendix B is a documented copy of the most complicated version of this model which was estimated.

3.4 Economic Welfare

The model is run for a forty-year period, during which demand grows constantly. A decision is made every year on whether or not to expand capacity on each of the competing modes or on some subset of them. The model also keeps track of the net present value of economic surplus less capital and maintenance cost over the entire time period. This net present value calculation can be compared for various pricing policies and investment criteria.

In order to decide whether or not to expand capacity of a given mode in a given year, the increase in economic surplus is compared with the increased capital and operating costs which would result. The increase in surplus due to the expansion of capacity in mode i is calculated as:

$$\Delta S_i = \left(\int_0^{Q_{t,(C+K)_i}} P(Q)dQ - \int_0^{Q_{t,C_i}} P(Q)dQ \right) -$$

$$\sum_j \left(\int_0^{Q_{jt,(C+K)_i}} MC_j dQ - \int_0^{Q_{jt,C_i}} MC_j dQ_j \right) \qquad (15)$$

where the subscript C_i indicates the capacity of mode i before expansion and $(C+K)_i$ its capacity after expansion, and where there are j competing modes. Capacity on mode i should be expanded when

$$\Delta S_i \geq rK_i + M_i \qquad (16)$$

where r is the social discount rate, K_i the capital cost of greater capacity for mode i, and M_i the increased annual operating cost, which is assumed to be independent of the level of traffic using the expanded mode i.

Because we are considering a number of modes in this model, we would calculate in each year not only whether each facility should be expanded separately, but whether the competing facilities should all be expanded. For example, if there were two competing facilities in the model, we would expand capacity when

$$\Delta S_i + \Delta S_j \geq r(K_i + K_j) + M_i + M_j \qquad (17)$$

The model was set up in such a way that it first determined whether all facilities should be expanded. If the result was that they should be, then they were, and the programme moved on to the next year. However, if the answer was that

all facilities should not be expanded together, then it checked the various subsets, in order to see whether any should be expanded. If there were cases where, for example, there were two competing modes, and the calculations said that either mode separately should be expanded, but that both together should not be, then a priority rule was established to indicate which mode would get priority.

Finally, the net present value of economic surplus less capital cost of any pricing and investment policy over a planning horizon of T years is equal to

$$\sum_{t=0}^{T} (1/(1+r)^t) \{ \int_0^{Q_t} P(Q_t)dQ_t - \sum_j (\int_0^{Q_{jt}} MC_j dQ_{jt} - M_{jt} - K_{jt}) \} \qquad (18)$$

In this formulation there are j different modes. It should also be noted that the K_{jt} are equal to zero except in those years when new capacity is actually added.

CHAPTER FOUR
APPLICATIONS OF THE MODEL

4.1 A Model of an International Airport

4.1.1 Specification

The model presented here is fundamentally the same model as that presented in my previous study (Borins, 1980). The major difference is that there are cross-elasticities of demand between peak and off-peak trips, which makes the demand functions for peak and off-peak periods interdependent. However, the capital expansion decision is made somewhat easier because the same capacity is used in both the peak and off-peak. There are competing modes in the sense that the two times of the day compete with each other, even though they are using the same physical facilities.

The choice of parameters for the original airport model is discussed in Borins (1980), pp. 47-57, and will not be repeated here. Table 1 presents the basic data used in the airport model.

The airport model is written in terms of aircraft operations, rather than passenger trips, as was the case with the model in Borins (1980). The values of passengers' time that were used were the high estimates in the previous report. Only one value of passengers' time was used because the previous report found that the results were not sensitive to the assumptions made about it.

Elasticities enter into this model in two different ways, in the trans-log demand functions and in the overall demand function. A number of different values were used, since the previous report found that the results were sensitive to the elasticities used.

Table 1. Basic Data for Airport Model

Capital Cost of Initial Airport Capacity	$800,000,000
Annual Operating and Maintenance Cost of Initial Airport Capacity	$ 30,000,000
Size of Airport Capacity Units	40 aircraft/hr
Annual Rate of Growth of Demand	5%
Value of Passengers' Time	$11.50/hr
Aircraft Load Factor	65% of seats
Length of Periods:	
-- peak period	6 hrs/day
-- off-peak period	10 hrs/day
F_i (resource costs independent of congestion):	
-- peak period	$24,500/flight
-- off-peak period	$23,000/flight
Existing User Fees, peak and off-peak periods	$ 1,100/flight
Airport Capacity during peak period (40 flights x 6 hrs)	240 flights
Airport Capacity during off-peak period (40 flights x 10 hrs)	400 flights

As discussed in Hogan (1980), there are a number of restrictions on the elasticities of the trans-log functions.

The B_i's, which represent the initial value shares of the different modes, must sum up the unity.

The cross-elasticities are symmetric, that is, $C_{ij}=C_{ji}$. Furthermore, as we assume that a proportional change in all user costs does not change the value shares, we then find

that

$$\sum_j C_{ij} = 0.$$

These two restrictions mean that, for the airport model,

$$C_{ij} = C_{ji} = -C_{ii} = -C_{jj}.$$

Finally, values of A for the trans-log function and a_0 in the overall demand function for trips were specified. To represent the growth in traffic over time, the value of a_0 was increased by 5% per annum.

For the airport model, the parameter x of the cost function was 1 (see equation 9), which is what would be found in steady-state queuing models. Such models have been found accurately to represent delays experienced by aircraft (Federal Aviation Administration, 1969; Federal Aviation Administration, 1975; Horonjeff, 1962). In this case, x then refers to the parameter of the cost function (12) which describes the pricing policy in force. For x=2, there is marginal cost pricing. For x=1, there would be average cost pricing. Existing pricing policies would have x=1 as well as a toll of t_i (as shown in equation 8).

4.1.2 Results

In the tables that follow, A_0 is the parameter of the overall demand function, and A1 its elasticity. Values for the B's and C's in the trans-log function are also shown. Constant Returns to scale refers to those cases where the capital costs were $800,000,000 for all units of capacity, and operating and maintenance costs $30 million per year. Other cases are shown where capital and O & M costs either increase or decrease by 20% per increment. For the former,

capital costs of the second increment would be $960,000,000 while for the latter they would be $640,000,000. This was done to see the effects of both increasing and decreasing returns on the outcomes.

X is the parameter of the cost function, and TOLL indicates the toll charged. It is in thousands of dollars. RR is the interest rate used for capacity expansion decisions. The social opportunity cost of capital is assumed to be 10%, so an RR of .1 would indicate that capital was being charged its proper opportunity cost. NPVR is the net present value of the revenue stream, in millions of dollars. It is equal to the tolls charged minus capital and operating and maintenance costs. NPVS is the net present value of economic surplus, also in millions of dollars. DEV (%) is the relative deviation from the value of economic surplus with marginal cost pricing, and an interest rate of 10%, which produces the maximum level of economic surplus. DEV is the absolute deviation from the optimum, measured in millions of dollars. CAP (PEAK/OFF) is the ultimate capacity of the airport by the end of the run, in terms of aircraft operations in the peak and off-peak periods.

In inspecting all the tables, one major result is clear. The relative deviations from the marginal cost pricing-social opportunity cost investment optimum are very small, and in no case larger than 1.5%. This confirms the result of the simple models using only one facility. The welfare surface is relatively flat around the optimum for a wide range of non-optimal pricing and investment policies. However, the absolute losses are large enough so as not to be dismissed out of hand. The welfare loss due to existing pricing policies, assuming that policy-makers use a social

Table 2. Results of Airport Model: High Elasticities and Cross-Elasticities, Constant Returns to Scale

Values of Parameters:

A_0 = 20000. A_1 = -1.5000
B_1 = 0.6330 B_2 = 0.3670
C_{11} = -.7500 C_{12} = 0.7500 C_{21} = 0.7500 C_{22} = -.7500

Constant Returns to Scale for Capital and Operating Costs

X	TOLL	RR	NPVR ($ x 10^6)	NPVS ($ x 10^6)	DEV (%)	DEV ($ x 10^6)	CAP (PEAK/OFF)
2.00	0.00	0.005	-1181.9	40949.8	0.51	210.5	960/1600
1.00	0.00	0.005	-1906.4	40830.8	0.80	329.5	1200/2000
1.00	1.00	0.005	- 796.4	40876.0	0.69	284.3	1200/2000
1.00	3.00	0.005	1096.8	40815.4	0.83	344.9	960/1600
1.00	5.00	0.005	2620.6	40570.0	1.43	590.3	960/1600
2.00	0.00	0.050	- 639.0	41111.6	0.11	48.7	960/1600
1.00	0.00	0.050	-1608.0	41017.2	0.34	143.1	960/1600
1.00	1.00	0.050	- 521.7	41051.7	0.26	108.6	960/1600
1.00	3.00	0.050	1321.2	40948.7	0.51	211.6	960/1600
1.00	5.00	0.050	2804.8	40696.3	1.12	464.0	720/1200
2.00	0.00	0.100	- 353.3	41160.3	0.00	0	720/1200
1.00	0.00	0.100	-1484.8	41040.7	0.29	119.6	960/1600
1.00	1.00	0.100	- 375.9	41104.4	0.13	55.9	720/1200
1.00	3.00	0.100	1411.4	40993.4	0.40	166.9	720/1200
1.00	5.00	0.100	2869.9	40710.0	1.09	450.3	720/1200
2.00	0.00	0.150	- 210.0	41150.1	0.02	10.2	720/1200
1.00	0.00	0.150	-1387.8	41058.5	0.24	101.8	720/1200
1.00	1.00	0.150	- 331.7	41091.2	0.16	69.1	720/1200
1.00	3.00	0.150	1452.8	40984.0	0.42	176.3	720/1200
1.00	5.00	0.150	2896.8	40670.0	1.11	490.3	720/1200
2.00	0.00	0.500	349.1	41029.9	0.31	130.4	480 / 800
1.00	0.00	0.500	-1222.8	40870.2	0.70	290.1	480 / 800
1.00	1.00	0.500	- 191.9	40914.9	0.59	245.4	480 / 800
1.00	3.00	0.500	1554.7	40855.2	0.74	305.1	480 / 800
1.00	5.00	0.500	2968.7	40612.7	1.33	547.6	480 / 800

See text for an explanation of the terms used.

Table 3. Results of Airport Model: High Elasticity, Low Cross-Elasticity, Constant Returns to Scale

Values of Parameters:

A_0 = 20000. A_1 = -1.5000
B_1 = 0.6780 B_2 = 0.3220
C_{11} = -.2500 C_{12} = 0.2500 C_{21} = 0.2500 C_{22} = -.2500

Constant Returns to Scale (Capital and Operating Costs Unchanged for all Increments)

X	TOLL	RR	NPVR ($ x 10^6)	NPVS ($ x 10^6)	DEV (%)	DEV ($ x 10^6)	CAP (PEAK/OFF)
2.00	0.00	0.005	-1295.9	40772.1	0.50	206	1440/2400
1.00	0.00	0.005	-1971.7	40690.2	0.70	288	1680/2800
1.00	1.00	0.005	- 834.7	40750.9	0.53	217.2	1440/2400
1.00	3.00	0.005	1058.3	40593.4	0.69	284.8	1440/2400
1.00	5.00	0.005	2550.6	40435.5	1.32	542.7	1200/2000
2.00	0.00	0.050	- 686.6	40946.6	0.07	31.6	1200/2000
1.00	0.00	0.050	-1663.7	40872.6	0.25	105.6	1200/2000
1.00	1.00	0.050	- 573.7	40911.4	0.16	66.8	1200/2000
1.00	3.00	0.050	1264.5	40813.8	0.40	164.3	1200/2000
1.00	5.00	0.050	2733.9	40552.9	1.03	425.3	960/1600
2.00	0.00	0.100	- 342.2	40978.2	0.00	0	960/1600
1.00	0.00	0.100	-1533.8	40891.0	0.21	87.2	1200/2000
1.00	1.00	0.100	- 484.3	40927.5	0.12	50.7	1200/2000
1.00	3.00	0.100	1346.4	40840.4	0.33	137.8	960/1600
1.00	5.00	0.100	2806.5	40566.1	1.00	412.0	960/1600
2.00	0.00	0.150	- 142.3	40963.3	0.03	14.8	960/1600
1.00	0.00	0.150	-1467.9	40893.9	0.20	84.4	960/1600
1.00	1.00	0.150	- 404.8	40928.5	0.12	49.6	960/1600
1.00	3.00	0.150	1394.0	40823.4	0.37	154.7	960/1600
1.00	5.00	0.150	2825.3	40559.7	1.02	418.4	960/1600
2.00	0.00	0.500	593.1	40764.8	0.52	213.3	720/1200
1.00	0.00	0.500	-1315.9	40664.9	0.76	313.2	960/1600
1.00	1.00	0.500	- 268.7	40721.2	0.62	256.9	720/1200
1.00	3.00	0.500	1474.0	40687.3	0.70	290.8	720/1200
1.00	5.00	0.500	2890.3	40455.0	1.27	523.1	720/1200

Table 4. Results of Airport Model: Low Elasticity, Cross-Elasticities, Constant Returns to Scale

Values of Parameters:

$A_0 = 3000.$ $A_1 = -0.7500$
$B_1 = 0.6890$ $B_2 = 0.3110$
$C_{11} = -.2000$ $C_{12} = 0.2000$ $C_{21} = 0.2000$ $C_{22} = -.2000$

X	TOLL	RR	NPVR ($\$ \times 10^6$)	NPVS ($\$ \times 10^6$)	DEV (%)	DEV ($\$ \times 10^6$)	CAP (PEAK/OFF)
2.00	0.00	0.005	-2314.7	943465.4	0.04	357.8	2400/4000
1.00	0.00	0.005	-3350.6	943399.7	0.04	423.5	2400/4000
1.00	5.00	0.005	4899.3	943088.5	0.08	734.7	2400/4000
2.00	0.00	0.080	- 234.4	943825.5	0.00	0	1920/3200
1.00	0.00	0.080	-2681.0	943688.7	0.01	134.5	2160/3600
1.00	5.00	0.080	5619.1	943451.1	0.04	372.1	1920/3200
2.00	0.00	0.090	- 192.4	943825.9	0.00	0	1920/3200
1.00	0.00	0.090	-2545.4	943710.1	0.01	113.1	1920/3200
1.00	5.00	0.090	5706.4	943456.0	0.04	367.3	1920/3200
2.00	0.00	0.100	199.5	943823.3	0.00	0	1920/3200
1.00	0.00	0.100	-2537.5	943710.0	0.01	113.3	1920/3200
1.00	5.00	0.100	5712.2	943455.9	0.04	367.4	1920/3200
2.00	0.00	0.110	216.9	943822.7	0.00	0.6	1920/3200
1.00	0.00	0.110	-2518.5	943708.1	0.12	115.1	1920/3200
1.00	5.00	0.110	5742.2	943470.6	0.04	352.7	1680/2800
2.00	0.00	0.120	536.2	943806.9	0.00	16.3	1920/3200
1.00	0.00	0.120	-2411.9	943687.1	0.01	136.1	1920/3200
1.00	5.00	0.120	5819.3	943453.6	0.04	369.6	1680/2800
2.00	0.00	0.150	1005.1	943784.4	0.00	38.9	1680/2800
1.00	0.00	0.150	-2387.5	943678.8	0.02	144.5	1920/3200
1.00	5.00	0.150	5834.9	943447.7	0.04	375.5	1680/2800
2.00	0.00	0.900	5980.0	942353.9	0.16	1469.4	960/1600
1.00	0.00	0.900	-1702.7	942203.5	0.17	1619.7	1440/2400
1.00	5.00	0.900	6206.7	942369.0	0.15	1454.2	1200/2000

Table 5. Results of Airport Model: High Elasticity, High Cross-Elasticity, Economies of Scale

Values of Parameters:

A_0 = 20000. A_1 = -1.5000
B_1 = 0.6330 B_2 = 0.3670
C_{11}= -.7500 C_{12}= 0.7500 C_{21}= 0.7500 C_{22}= -.7500

Capital and Operating Costs Decrease by 20% per Increment

X	TOLL	RR	NPVR ($\$ \times 10^6$)	NPVS ($\$ \times 10^6$)	DEV (%)	DEV ($\$ \times 10^6$)	CAP (PEAK/OFF)
2.00	0.00	0.005	-1202.2	41234.3	0.25	103.2	2400/4000
1.00	0.00	0.005	-1649.8	41199.3	0.33	138.3	2400/4000
1.00	1.00	0.005	- 545.2	41220.6	0.28	116.9	2400/4000
1.00	3.00	0.005	1278.5	41059.1	0.65	268.4	2400/4000
1.00	5.00	0.005	2771.6	40776.9	1.36	560.6	2400/4000
2.00	0.00	0.090	- 662.7	41334.3	0.01	3.2	1680/2800
1.00	0.00	0.090	-1401.0	41302.2	0.09	35.4	1680/2800
1.00	1.00	0.090	- 322.1	41314.2	0.06	23.4	1440/2400
1.00	3.00	0.090	1475.0	41163.9	0.42	173.6	1440/2400
1.00	5.00	0.090	2928.1	40855.0	1.17	482.6	1200/2000
2.00	0.00	0.100	- 649.2	41337.6	0.00	0	1440/2400
1.00	0.00	0.100	-1392.1	41301.8	0.09	35.7	1680/2800
1.00	1.00	0.100	- 317.6	41313.9	0.06	23.7	1440/2400
1.00	3.00	0.100	1492.7	41167.6	0.41	170.0	1200/2000
1.00	5.00	0.100	2932.2	40854.7	1.17	482.8	1200/2000
2.00	0.00	0.110	- 587.8	41335.1	0.02	2.4	1440/2400
1.00	0.00	0.110	-1370.6	41303.1	0.08	34.4	1440/2400
1.00	1.00	0.110	- 314.5	41313.6	0.06	24.0	1440/2400
1.00	3.00	0.110	1496.0	41167.0	0.41	170.5	1200/2000
1.00	5.00	0.110	2942.0	40853.2	1.17	484.3	1200/2000
2.00	0.00	0.500	18.4	41215.3	0.30	122.3	720/1200
1.00	0.00	0.500	-1235.8	41157.9	0.41	169.7	960/1600
1.00	1.00	0.500	- 190.1	41180.2	0.38	157.4	960/1600
1.00	3.00	0.500	1578.7	41073.1	0.64	264.5	720/1200
1.00	5.00	0.500	2998.9	40784.1	1.34	553.5	720/1200

Table 6. Results of Airport Model: High Elasticity, Low Cross-Elasticity, Diseconomies of Scale

Values of Parameters:

$A_0 = 20000.$ $A_1 = -1.5000$
$B_1 = 0.6780$ $B_2 = 0.3220$
$C_{11} = -.2500$ $C_{12} = 0.2500$ $C_{21} = 0.2500$ $C_{22} = -.2500$

Capital and Operating Costs Increase by 20% per Increment

X	TOLL	RR	NPVR ($\$ \times 10^6$)	NPVS ($\$ \times 10^6$)	DEV (%)	DEV ($\$ \times 10^6$)	CAP (PEAK/OFF)
2.00	0.00	0.005	-1284.1	40581.5	0.63	256.6	1200/2000
1.00	0.00	0.005	-2160.1	40448.1	0.95	389.9	1200/2000
2.00	0.00	0.050	- 557.4	40817.1	0.05	20.8	960/1600
1.00	0.00	0.050	-1723.0	40709.9	0.31	128.0	960/1600
2.00	0.00	0.100	- 215.2	40837.9	0.00	0	960/1600
1.00	0.00	0.100	-1613.9	40724.4	0.28	113.6	960/1600
2.00	0.00	0.150	86.5	40850.2	-0.03	0	720/1200
1.00	0.00	0.150	-1533.4	40702.0	0.33	136.0	960/1600
2.00	0.00	0.500	761.9	40661.0	0.43	177.0	480/ 800
1.00	0.00	0.500	-1317.6	40446.5	0.96	391.4	720/1200

opportunity cost of capital of 10%, is about $ 50 million over the 40-year planning horizon of the model. This welfare loss is roughly equivalent to an annual loss of $ 5 million. The absolute welfare loss resulting from larger deviations is more substantial. Social opportunity costs of capital of either .5% or 50%, even combined with marginal cost pricing, lead to welfare losses of $200 million, and a pricing policy of $5,000 per aircraft operation, both at the peak and off-peak, generally leads to a welfare loss with a net present value of $500,000,000 which is quite substantial. By and large, social opportunity cost of capital estimates between 5% and

15% affect the level of welfare losses very little.

The absolute welfare losses are roughly similar for all the assumptions about the overall elasticity of demand and the cross-elasticities. However, higher elasticities do lead to high relative welfare losses. This is because the demand functions approach the vertical axis less rapidly for lower elasticities than for higher elasticities, so that the area under the demand curve is larger, so that the loss relative to the area under the demand curve is smaller. Comparing Tables 2 and 3 shows that higher cross-elasticities lead to slightly higher deviations. Table 4 decreases the overall elasticity from 1.5 to .75, while maintaining low cross-elasticities. In this case, the relative deviations are much smaller. This confirms the result of the simpler models that lower elasticities lead to lower relative deviations.

These models show an application of the economics of the second best discussed in the previous report. It was shown in Borins (1981) that, if the facility manager charges a constant toll over the life of the facility, he can achieve a smaller relative deviation than if there were average cost pricing. However, as the constant toll increases, the relative deviation becomes larger than would be the case for average cost pricing. This can be seen in any of the tables. For example, in Table 2, for an interest of 10%, the deviation for average cost pricing (without any toll) is .29%. For a toll of $1,000 it goes down to .136%, while for a toll of $3,000, it increases to .41%.

Tables 2, 3, and 4 all illustrate the effects of the interest rate and pricing policy on the amount of capacity ultimately required. Low interest rates lead to more

capacity than high interest rates. Average cost pricing leads to more capacity than marginal cost pricing, while substantial tolls lead to less capacity than low tolls. The differences between the highest (50%) and the lowest (.5%) interest rates are quite pronounced, as the ultimate capacity is twice as large for the latter than the former. However, for the range of discount rates which would commonly be used by decision makers (5% to 15%) ultimate capacities are often the same for all three, or differ only by one increment. (Of course, the capacity would be added sooner for the lower rates, which is not shown in these tables.)

We can also look at the implications of this model for financial self-sufficiency. In all cases, as the interest rate used to make capacity expansion decisions is increased, the project comes closer to financial self-sufficiency. This is to be expected, since less capacity is provided. As the demand becomes less elastic, the projects come closer to achieving financial self-sufficiency. This is because, with lower elasticities, user fees discourage less traffic and therefore bring in more revenue.

The previous report showed that the simple model supported Starrett's (1978) theoretical conclusion that marginal cost pricing (with investment determined by the social opportunity cost of capital) would pay for all but the initial increment in capacity. In this more complicated model, Starrett's result does not hold. Even for the high elasticity result in Table 2, the deficit for marginal cost pricing is $353,287,000, which means that about 55 percent of the cost of the initial capacity has been paid. Table 4 indicates that, when the elasticity and cross-elasticity are low, the project can break even for marginal cost pricing,

as it brings in a profit of $199,528,000. Tables 2 and 3 also show that for interest rates of 5 to 15% the airport loses money with the existing pricing policy (tolls of $1,000 per flight). However, with tolls of $3,000 per flight it breaks even handily. Thus, somewhat less than a doubling of present airport user fees would allow it to break even. This would result in a loss of economic surplus of about $100 million, so that the increased welfare loss (over existing policies) of a break-even policy would be about $50 million.

This raises the issue of equity versus efficiency. If government decides that it is equitable to make the users of the airport pay for its capital cost, even if it is by means of a non-optimal pricing policy assessed on flights at all times of day rather than a marginal cost pricing policy, the relative loss in efficiency is minimal, while the absolute loss, though noticeable, is not great.

Tables 5 and 6 can be used to investigate the impacts of increasing and decreasing returns to scale in operating and capital costs. Table 5 should be compared to Table 2 and Table 6 to Table 3. Comparing Tables 2 and 5, the relative losses in economic surplus are identical in both cases. With increasing returns, the ultimate capacity is greater, as would be expected. At low interest rates, the result is most dramatic, with the ultimate facility being more than twice as big when capital and operating costs decrease. Higher interests lead to a smaller differential between the two tables. Comparing the revenue results, marginal cost pricing leads to greater deficits when there are increasing returns to scale, but average cost pricing as well as tolls come closer to self-sufficiency. In either case, however,

the results are quite similar. Decreasing costs have two contradictory effects. On the one hand, expansion comes sooner, which increases costs. On the other hand, costs per unit are smaller.

Tables 3 and 6 show the impact of capital and operating costs that increase by 20% per increment. The welfare losses are about the same in both cases. The ultimate capacity of the airport is somewhat smaller when costs increase by 20% per increment. The impact on the net present value of revenues is the opposite of the previous case. Deficits are smaller under marginal cost pricing with decreasing returns to scale than with constant returns to scale. Under average cost pricing deficits are greater. However, the results in both cases are close to those for constant returns to scale.

All told, then, the airport model, now specified with interdependent demand functions, provided few differences from the previous simpler model. For a range within the vicinity of the marginal cost pricing and social opportunity cost of capital optimum the welfare surface is quite flat, with small relative welfare losses and acceptable, though non-negligible, absolute welfare losses. This range can be taken to lie between social opportunity cost rates of interest of 5 and 15%, and pricing policies ranging from a totally free airport to one with fees, assessed at all times of day, up to $2,000 per flights, which is roughly double the present level. Within that range, welfare losses will be less than $100,000,000 for the forty-year planning horizon, or $10 million a year. Within this range, government could use pricing and investment policies to pursue other objectives, such as financial self-sufficiency, or

minimization of the capital budget for the airport, with little, or at most tolerable, loss of economic efficiency. The next section of this result discusses whether these results still hold for a larger, more complicated urban transportation model.

4.2 An Urban Transportation Model

4.2.1 Specification

The urban transportation model involves two competing transportation facilities, an expressway and a subway, which run along the same route. They are both 10 miles in length, and carry passengers between the city centre and the suburbs. In addition to substitution between the two facilities, there is inter-temporal substitution between the peak and off-peak periods. Thus, the model consists of four substitutable facilities: the expressway during the peak and off-peak, and the subway during the peak and off-peak. Because the model assumes that all the morning peak trips will be from the suburbs to the central city and that the afternoon peak trips are in the opposite direction, peak capacity must be measured in each direction. Thus, the model deals with trips in only one direction, and the results can be doubled to refer to both directions. For example, if tolls assessed on users travelling in one direction can cover one-half of the capital cost of the subway, then tolls assessed on users travelling in both directions will cover the entire capital cost.

This section discusses in detail how the various parameters of the model were calculated, based on data derived from Toronto. The model should not be thought of as a model

of the Toronto transportation system as much as a model of a feasible transportation system, based on data from Toronto. In terms of the analogy to Toronto, however, the model could be considered to represent each of the various corridors between the suburbs and the central city which are served by expressways and rapid transit facilities which run roughly parallel to each other. These would be: the GO train and the Gardiner expressway, approaching downtown Toronto from either the east or the west, the Spadina expressway and the Spadina extension of the subway system, and the Don Valley Parkway and the main north-south line of the Toronto subway.

4.2.1.1 Parameters of the Congestion Cost Function

The parameter x of the cost function (see equation 9) should be quite small, probably around .2 to .4. The empirical research has shown that on expressways and subways traffic moves at close to the intended speed until volume is near design capacity (see Keeler and Small, 1977 and Boardman and Lave, 1977).

The Z terms in the cost function are the parameters which represent the cost of a trip using a given facility, taken by an average user, when the facility is operating without any congestion. The model is specified in person trips, rather than vehicle trips, so as to make the expressway and the subway comparable.

The four values of Z are calculated as follows:

Expressway Peak Hour Trips (Z_1)

Time component:

\$5 per person per hour X 10 miles at 60 m.p.h. = \$ .83

Gasoline:

$1.75 per gallon X 10 miles at 17 m.p. gallon ÷ 1.5 persons per car = $ .69

Therefore, time and gasoline cost = $1.50 per person.

Expressway Off-Peak Hour Trips (Z_2)

Time Component:

$3 per person per hour X 10 miles at 60 m.p.h. = $ .50

Gasoline:

$1.75 per gallon X 10 miles at 17 m.p.g. ÷ 1.7 persons per car = $ .61

Therefore, time and gasoline cost = $1.10 per person.

Subway Peak Hour Trips (Z_3)

Time Component:

$3 per person per hour X 10 miles at 30 m.p.h. = $1.00/pers

Subway Off-Peak Trips (Z_4)

Time Component:

$1.50/person per hour X 10 miles at 30 m.p.h. = $ .50/pers

In these calculations, several things should be noted. Following most empirical research on the valuation of urban travel time, we have set drivers' time at a higher value than public transit users' time, and peak hour passengers' time at a higher value than off-peak hour passengers' time. The automobile usage and gasoline mileage figures are based on studies by the Ontario Ministry of Transportation and Communications (see Borins, 1980, p. 92). It is assumed that the subway averages 30 miles per hour because it makes

frequent stops. Subway passengers do not pay the operating cost of the the train because that cost would be incurred whether or not they travelled. Those costs are considered as part of the operating and maintenance cost of the subway system, which will be discussed below.

The cost of $1.75 per gallon is considered to represent the real resource cost of gasoline, in 1980 dollars, which is based on the assumption that the OPEC oil price is close to, or perhaps slightly higher than, the long-run opportunity cost of alternative sources of oil or other energy sources. Part of this $1.75 per gallon is accounted for in provincial gasoline taxes, which are often considered to be road taxes (see, for example, Sherman, 1971 and 1972). In this model, the provincial gasoline tax is not treated as a road tax, for two reasons. First, it goes into general tax revenues, rather than being earmarked for road user fees. Secondly, since the price of Canadian gasoline, exclusive of the provincial tax, is lower than the opportunity cost, the provincial tax can be considered as part of the opportunity cost, to make up for the implicit subsidy.

The next set of parameters calculated were the F's, which are the resource costs incurred by expressway and subway passengers independent of congestion on the expressway or subway. For expressway travellers, this would include the car trip between home and the expressway and the trip between the expressway exits and their actual work-place, as well as parking. For subway passengers, this would include driving to the expressway and parking for some, and just walking for the rest.

Expressway Peak Hour Trips (F_1)

Time Cost:
$5/person/hour X 10 miles at 30 m.p.h. = $1.67/person

Gasoline Cost:
$1.75/gal X 10 miles at 17 m.p.g. ÷ 1.7 persons/car
= $ .69/person

Parking Cost:
$2/car ÷ 1.5 persons/car = $1.33/person

Therefore, time, gasoline, and parking costs total $3.70 per person.

Expressway Off-Peak Hour Trips (F_2)

Time Cost:
$3/person/hour X 10 miles at 30 m.p.h. = $1.00/person

Gasoline Cost:
$1.75/gal X 10 miles at 17 m.p.g. ÷ 1.7 persons/car
= $ .61/person

Parking Cost:
$2/car ÷ 1.7 persons/car = $1.18/person

Therefore, time, gasoline, and parking costs total $2.80 per person.

Subway Peak Hour Trips (F_3)

For those who drive to the subway, we have:

Time Cost:
$3/person/hour X 10 miles at 30 m.p.h. = $1.00/person

Parking Cost:
$1.00/person

Gasoline Cost:

$1.75/gallon X 10 miles at 17 m.p.g. = $1.03/person

Therefore, time, parking and gasoline costs total $3.03 per person.

In this calculation, we were not certain as to either the valuation of time or the number of passengers per car. The assumption of $3 per hour probably would bias the result downward, and the assumption of one passenger per car would bias it upward. The two biases will probably cancel out one another.

For those who do not drive to the subway, there is only walking time, at $3/person/hour X 40 minutes = $2.00.

This assumes they walk an average of 20 minutes to the subway and from the subway.

Finally, we assume that 25% of all subway passengers drive to the terminal, while the rest walk. Thus, the value of F_3 is (.25) $3.03 + (.75) $2.00 = $2.25/person.

Subway Off-Peak Hour Trips (F_4)

For those who drive to the subway, we have

$1.50/person/hour X 10 miles at 30 m.p.h. + $1.00 parking + $1.75/gal X 10 miles at 17 m.p.g. = $2.53/person.

For those who do not drive to the subway, we have

$1.50/hour X 40 minutes = $1.00.

Assuming that 25% drive and the rest walk, the value of F_4 is (.25) $2.53 + (.75) $1.00 = $1.40/person.

4.2.1.2 Capital, Operating and Maintenance Costs, and Capacity

Capital and operating and maintenance costs for the expressway were as calculated in Borins (1980), pp. 94 and 95. The capital cost for the first lane in either direction is $97,500,000 (including land), while the second lane would cost $4.25 million, the third lane $15 million, and all subsequent lanes $4.25 million. Operating and maintenance costs were $29,000 per annum for the first lane, $53,000 for a two-lane road, $136,000 for a three-lane road, $181,000 for a four-lane road, and increased by $45,000 per lane for each additional lane.

The maximum physical capacity of the expressway is 2,000 cars per lane per hour. Thus, its one-way capacity during a 2-hour peak would be 2,000 X 1.5 X 2 = 6,000 passengers per lane, while its one-way capacity during a 22-hour off-peak period would be 2,000 X 1.7 X 22 = 75,000 passengers.

The capital cost of the subway was assumed to be $40 million per mile. This is something of an upper bound estimate, based on recent experience in Toronto. The Bloor-Danforth subway was extended by 4 kilometres, at a cost of $140 million, or $35 million per kilometre or $22 million per mile (Soberman and Hazard, 1980, p. 101). Based on the 1978 purchase price of $390,000 per car paid by the Toronto Transit Commission, we assume that subway cars cost $400,000 each (Soberman and Hazard, 1980, p. 325). These cars can carry a maximum load of 320 people, and have a service life of 30 years.

The subway is assumed to start with 16 cars, costing

$400,000 each. The cost of this initial complement of cars is $6,400,000. The initial capital cost of the subway is 10 miles X $40,000,000 per mile ÷ 2 (for the one-way model) = $200,000,000. The initial capital cost of the subway is then $206,400,000.

Subway operating costs were assumed to be about $140,000 per car per annum. This was based on the 1979 Annual Report of the Toronto Transit Commission. Thirty percent of the TTC mileage is accounted for by the system's approximately 600 cars. Thus, the TTC's expenses (see p. 23 in the annual report) were prorated with 30% going to the subway, and these were then expressed on a per-car basis. The only expense not prorated was automotive fuel, which went entirely to buses, while electric tractic power was assumed to be charged entirely against the subway.

Finally, the capacity of the subway had to be determined. Unlike the road, determining the capacity of the subway system is a function of both the amount of track and the number of trains that are run. It was assumed that, at the outset, the subway system would run four-car trains every 10 minutes, during both the peak and the off-peak.

Since the peak period is two hours long, this would result in the following capacity in terms of the number of passengers:

6 trains/hour X 4 cars/train X 320 persons/car X 2 hours
= 15,300 passengers (or approximately 16,000).

During the 16-hour off-peak period the capacity would be:

6 trains/hour X 4 cars/train X 320 persons/car X 16 hours
= 123,000 passengers.

These capacity calculations assume that the subway

system is shut down for 6 hours, and that the traffic flows are relatively constant with both the peak and off-peak periods.

Table 7 indicates how capacity of the subway system is increased, first by running cars more frequently, and then by adding more cars to the system. While this could be done continuously, we preferred to work with discrete increments in capacity.

Table 7. Increasing the Capacity of the Subway

(table entries are in passengers per period)

Action	Peak Period Capacity	Off-Peak Period Capacity
Initial System	15,300	123,000
run peak period trains every 8 minutes	15 x 4 x 320 = 19,200	123,000
run peak period trains every 6 minutes, off-peak trains every 8 minutes	20 x 4 x 320 = 25,600	120 x 4 x 320 = 153,600
peak period trains every 4 minutes, off-peak trains every 6 minutes	30 x 4 x 320 = 38,400	160 x 4 x 320 = 204,800
peak period trains every 2 minutes, off-peak trains every 4 minutes	60 x 4 x 320 = 76,800	240 x 4 x 320 = 307,200
increase car lengths to 6 cars from 4 in peak, run 4 car trains every 2 minutes in off-peak	60 x 6 x 320 =115,200	480 x 4 x 320 = 614,400

Table entries calculated as:
trains/period x cars/train x persons/car = persons

In order to increase the capacity of the subway, the number of cars must also be increased. In addition, cars must be bought so that replacements are available, since not all will be operational at any point in time. It was assumed that the increase in the number of cars would be roughly proportional to the increase in the total number of passengers. Thus, when the peak period trains run every 8, rather than 10 minutes, two cars are added. When peak period trains run every 4 minutes and off-peak trains every 6 minutes, 10 cars are added to the fleet. Finally, when peak period trains run every 2 minutes and off-peak period trains every 4 minutes, another 28 cars are added to the fleet. The capital and operating costs of the subway system would increase in proportion to the increase in the number of cars.

4.2.1.3 Parameters of the Demand Function

The parameters of the demand function had to follow the constraints of the trans-log model, as discussed above. The B_i's sum up to unity: they are .6 for the expressway during the peak, .35 for the expressway during the off-peak, .13 for the subway during the peak, and -.08 for the subway during the off-peak. The cross-elasticities (the C_{ij}'s) are constrained in two ways: $C_{ij} = C_{ji}$ and $-C_{ii} = \sum_j C_{ji}$. The urban transit literature (see Kraft and Domencich, 1972) has reached the conclusion that these cross-elasticities are relatively low. In this model, we made them as high as we felt was possible, because the airport model indicated that high elasticities show larger deviations from the economic surplus optimum. Based on casual empiricism, we specified the highest cross-elasticity between subway and expressway at the peak, and the next highest between expressway at the peak and the off-peak.

The matrix of cross-elasticities is shown in Table 8 below.

Table 8. Matrix of Cross-Elasticities

	Expressway Peak	Expressway Off-Peak	Subway Peak	Subway Off-Peak
Expressway Peak	-.6	.2	.25	.15
Expressway Off-Peak	.2	-.5	.20	.10
Subway Peak	.25	.20	-.60	.15
Subway Off-Peak	.15	.10	.15	-.40

Finally, elasticities of .5 and .75 were used for the overall demand function. Because many of the trips in the model were work trips, we felt that the elasticities would be quite low.

4.2.2 Solving the Model

Solving this model presented somewhat more difficulty than the previous model. In particular, the model often did not solve for a combination of a high overall demand elasticity and low values of x, the parameter of the cost function. When x is very low, congestion does not become serious until the model gets very close to full capacity, and the solution procedure had difficulty working in that range. As a result, the smallest value of x that was used was .4. This enabled us to use the higher overall demand elasticity of .75.

The constants A of the trans-log function and a_0 of the overall demand function were specified as to keep the shares of trips on the various modes relatively plausible. In general, the model started with about 20% of total traffic using the expressway during the peak, 40% using the expressway during the off-peak, 15% the subway during the peak, and 25% the subway during the off-peak. Given the shortness of the peak period, it is clear that the facilities are more crowded then than during the off-peak.

Demand grew by increasing a_0, the constant term in the overall demand function, at the rate of 2.5% per annum, which represents the rate of growth in urban travel in relatively mature cities such as Toronto. This rate is substantially less than the 5% annual growth rate used in the airport model. It is designed to reflect the influence of increasing energy prices on an energy-intensive activity such as urban travel.

The model's capacity expansion algorithm was more complicated than that of the airport model. First, the model looked at whether expansion of both the expressway and the subway was justified. If it was, both were expanded, and the model went on to the next year. It then considered whether to expand the subway by itself and whether to expand the expressway by itself. In those cases where expanding either separately could be justified, but both could not be justified, the priority rule of expanding the expressway was used. However, this problem turned out to be hypothetical. When the model indicated that there was a need for expansion in a given year, it was clear which facility was to be expanded.

Finally, the model kept track of prices, quantities,

the net value of economic surplus in every year, and costs and revenues for both the subway and the expressway. Results will be discussed in the following section of this paper.

4.2.3 Urban Transportation Model: Results

The results for the urban transportation model are presented in Tables 9 to 13. Tables 9 to 12 present results for four different cases, and Table 13 draws comparisons among the cases. Table 10 is probably the most realistic table, in that the elasticity of the overall demand function and the parameter of the cost function is closest to that which actually would be observed. Tables 11 and 12 present results for high assumptions about the elasticity of the demand function. In order for the model to work for such high elasticities, it was necessary to use a very high parameter of the cost function (.8) which is quite unrealistic. Thus, Tables 11 and 12 are doubly unrealistic. They are, nonetheless, useful in indicating the impact of different elasticity estimates on the results.

Table 13 sums up most of the major results. The absolute deviations from the marginal cost pricing, social opportunity cost investment optimum are quite small, with the existing pricing policy (free expressway, 60¢ subway fee) resulting in deviation of about $15 - $20 million over the 40-year planning horizon, regardless of the assumptions made about the elasticity of the overall demand function and the parameter of the cost function. The relative welfare losses increase with the elasticity of the demand function, with the existing pricing policy incurring a negligible loss for an elasticity of .5 and a cost parameter of .4 to a loss of

Table 9. Results of Urban Transportation Model, Low Elasticity Case

Assumptions: Elasticity of Demand Function = -.50 Parameter of Cost Function = .4

Case	Economic Surplus ($ x 10^6)	% Dev from Opt (-) .0001%	X Cap peak PAX	Net Rev ($ x 10^6)	S Cap peak PAX	Net Rev ($ x 10^6)
MCP, r=10%	33,781,155.9	0	42,000	-40.7	26,500	-215.4
MCP, r=5%	33,781,155.7	-	60,000	-47.2	26,500	-175.8
MCP, r=15%	33,781,155.5	-	24,000	-38.8	40,000	-174.1
Free X/S, r=10%	33,781,152.9	-	42,000	-105.3	26,500	-222.1
Free X/S, r=5%	33,781,152.4	-	60,000	-112.5	26,500	-221.4
Free X/S, r=15%	33,781,152.0	-	36,000	-103.5	26,500	-222.8
Free X, 60¢ S, r=10%	33,781,136.7	-	48,000	-107.0	20,000	-286.6
25¢ X, 60¢ S	33,781,141.1	-	48,000	-81.8	20,000	-256.8
60¢ X, 60¢ S	33,781,143.6	-	48,000	-52.4	26,500	-221.2
30¢ X/S at peak, free at off-peak	33,781,155.1	-	42,000	-96.2	26,500	-213.5
90¢ X/S at peak, free at off-peak	33,781,150.0	-	24,000	-81.9	20,000	-206.4
$3.20 X, $1.42 S (break-even)	33.781,098.9	.0002	36,000	+82.0	40,000	-81.8
$2.47 X/S (break-even)	33,781,081.9	.0002	54,000	+108.0	20,000	-107.6

Note: Interest rate used for capacity expansion decision is 10% unless otherwise indicated.

Legend: Dev - deviation Opt - optimum X - expressway S - subway Rev - revenue Pax - passengers
MCP - marginal cost pricing r - interest rate Cap - capacity Max - maximum

Table 10. Results of Urban Transportation Model, Medium Elasticity Case

Elasticity of Demand Functions = 0.75 Parameter of Cost Function = .4

Case	Economic Surplus ($ x 10)	% Dev from Opt	Expressway		Subway	
			Cap peak Pax	Net Rev ($ x 10)	Cap peak Pax	Net Rev ($ x 10)
MCP	74,389.4	0	42,000	-39.3	26,500	-173.6
MCP (Max x Cap = 12000 peak Pax)	74,389.3	-	12,000	-37.0	26,500	-173.0
Free X/S	74,386.0	-	42,000	-105.4	26,500	-222.1
Free X, 60¢ S	74,369.9	.03	54,000	-107.1	20,000	-179.2
Free X, 60¢ S (Max x Cap = 24000 peak Pax)	74,369.3	.03	24,000	-105.6	26,500	-179.0
60¢ X, 60¢ S	74,376.1	.02	48,000	-53.0	26,500	-169.5
25¢ X, 60¢ S	74,374.1	.02	48,000	-81.5	20,000	-256.5
$2.86 X, $2.86 S (break-even)	74,286.6	.14	48,000	+115.2	20,000	-114.8
30¢ X/S at peak, free at off-peak	74,388.5	-	42,000	-96.0	26,500	-213.3
60¢ X/S at peak, free at off-peak	74,387.4	-	42,000	-88.3	26,500	-208.0
90¢ X/S at peak free at off-peak	74,383.5	-	36,000	-81.7	20,000	-206.2

Note: Interest rate used for capacity expansion is 10% in all cases.
(-) less than .0001%

Legend: Dev - deviation Opt - optimum X - expressway S - subway Rev - revenue Pax - Passengers
MCP - marginal cost pricing r - interest rate Cap - capacity Max - maximum

Table 11. Results of Urban Transportation Model, High Elasticity Case

Assumptions: Elasticity of Demand Function = -1.25 Parameter of Cost Function = .8

Case	Economic Surplus ($ x 10)	% Dev from Opt	Expressway		Subway	
			Cap peak Pax	Net Rev ($ x 10)	Cap peak Pax	Net Rev ($ x 10)
MCP	2,145.3	0	54,000	+18.6	26,500	-133.2
Free X/S	2,139.1	.3	60,000	-108.7	26,500	-222.9
Free X, 60¢ S	2,126.6	.9	66,000	-109.6	26,500	-180.8
25¢ X, 60¢ S	2,130.2	.7	60,000	-85.4	26,500	-178.0
60¢ X, 60¢ S	2,130.7	.7	60,000	-58.3	26,500	-174.0
30¢ X/S at peak, free at off-peak	2,143.1	.1	60,000	-99.8	26,500	-214.9
60¢ X/S at peak, free at off-peak	2,143.7	.07	54,000	-92.2	26,500	-209.9
90¢ X/S at peak, free at off-peak	2,141.3	.2	48,000	-85.8	26,500	-207.7
$5.85 X/S (break-even)	1,896.6	11.6	48,000	+146.2	26,500	-146.0

Note: Interest rate used for capacity expansion is 10% in all cases.

Legend: Dev - deviation Opt - optimum X - expressway S - subway Rev - revenue Pax - Passengers
MCP - marginal cost pricing r - interest rate Cap - capacity Max - maximum

Table 12. Results of Urban Transportation Model, Highest Elasticity Case

Assumptions: Elasticity of Demand Function = -1.50 Parameter of Cost Function = .8

Case	Economic Surplus ($ x 10)	% Dev from Opt	Expressway		Subway	
			Cap peak Pax	Net Rev ($ x 10)	Cap Peak Pax	Net Rev ($ x 10)
MCP	912.9	0	48,000	+4.0	26,500	-143.9
Free X/S	907.4	.6	54,000	-106.2	26,500	-222.2
Free X, 60¢ S	896.1	1.8	54,000	-106.6	26,500	-185.1
25¢ X, 60¢ S	899.1	1.5	54,000	-85.4	26,500	-182.7
60¢ X, 60¢ S	899.5	1.5	48,000	-62.5	26,500	-179.7
30¢ X/S at peak, free at off-peak	911.1	.2	48,000	-98.3	26,500	-214.9
60¢ X/S at peak, free at off-peak	911.4	.2	48,000	-91.8	26,500	-210.6
90¢ X/S at peak, free at off-peak	909.1	.4	42,000	-86.1	26,500	-208.7

Note: Interest rate used for capacity expansion is 10% in all cases.

Legend: Dev - deviation Opt - optimum X - expressway S - subway Rev - revenue Pax - Passengers
MCP - marginal cost pricing r - interest rate Cap - capacity Max - maximum

Table 13. Absolute and Relative Welfare Losses

Nature of Deviation	elasticity=.50 cost parameter=.4 $ x 10^6	%	elasticity=.75 cost parameter=.4 $ x 10^6	%	elasticity=1.25 cost parameter=.8 $ x 10^6	%	elasticity=1.5 cost parameter=.8 $ x 10^6	%
free expressway, subway	3.0	-	3.4	-	6.2	.3	5.5	.6
free expressway, 60¢ subway fee	19.2	-	19.5	.03	19.7	.9	16.8	1.8
25¢ expressway fee, 60¢ subway fee	14.8	-	15.3	.02	15.1	.7	13.8	1.5
60¢ expressway fee, 60¢ subway fee	12.3	-	13.2	.02	14.6	.7	13.4	1.5
30¢ fee for both at peak, free at off-peak	.8	-	.9	-	2.2	.1	1.8	.2
60¢ fee for both at peak, free at off-peak	2.1	-	2.0	-	1.6	.07	1.5	.2
90¢ fee for both at peak, free at off-peak	5.9	-	5.9	-	4.0	.2	3.8	.4
break-even pricing (same fee for both subway and expressway)	74 (fee of $2.47)	.002	102.8 (fee of $2.86)	.14	248.7 (fee of $5.85)	11.6	NA	NA

1.8% for the highest elasticity. The other pricing policies show that absolute welfare losses do not vary with the elasticity and shape of the cost function but that relative welfare losses do vary with the elasticity of the demand function. The absolute welfare losses are smaller than those incurred in the airport model, which is consistent with the results of the previous study.

Table 13 also illustrates the application of the economics of the second best to intermodal competition in transportation. The existing pricing policy can be improved upon in one of either two ways, by reducing subway fees to zero, or by increasing the expressway fee until it is comparable to the subway fee. Of the two, the former brings the result closer to the marginal cost pricing optimum.

The table also shows the results of three policies designed to roughly approximate marginal cost pricing, namely charging fees of 30¢, 60¢, or 90¢ for both modes in the peak period. These policies, particularly the 30¢ peak-period fee, bring the outcome quite close to the marginal cost pricing optimum. It is also interesting to observe that the 60¢ and 90¢ peak-period fees do better than the 60¢ fee charged for both modes during both the peak and off-peak.

Table 9 illustrates the results of deviations from the correct social opportunity cost of capital. Using interest rates of 5% and 15% to determine capacity expansion dates leads to negligible welfare losses of less than one million dollars, in both cases of marginal cost pricing and free subways and expressways. This is similar to previous results both for the airport model in this report, and for the two simpler models in the previous report.

Table 10 illustrates another type of deviation that was

experimented with. It is clear from Tables 9 to 12 that the model tends to increase capacity on the expressway system much more rapidly than on the subway. Two cases were developed where the expressway was held at a capacity far below the equilibrium which would occur if it were allowed to expand. One could imagine that this occurred as a result of political opposition which made the expansion of the expressway impossible. In terms of the model, this was done simply by setting the capital cost of the next increment of expressway capacity beyond its constrained level equal to infinity. The second case in Table 10 shows what happens when, given marginal cost pricing, the expressway was held to two lanes, rather than the seven which would have otherwise been built. The fifth line shows what would happen when, given existing policies, capacity was held to four lanes, rather than nine. In both cases, the absolute welfare costs are miniscule, less than one million dollars. Thus, we can conclude that this constraint has a negligible effect on economic welfare.

The model was used to examine what would be required for the transportation system to break even. Tables 9 through 12 show that in all cases the subway has larger deficits than the expressway. There are an infinity of break-even policies: the table presents results for one of the simplest, which is to charge constant fees on all modes at all times. In this case, breakeven was defined to refer to the total system, so that profits on the expressway subsidize the remaining deficits on the subway. Attempts to find cases where both the subway and expressway broke even independently were unsuccessful: the subway fees would have had to have been raised to such a level that the model would not converge. Similarly, break-even policies where fees were

charged only during peak periods had the same result. The break-even policies that actually were simulated have some interesting results. As the elasticity of the demand function increases, the size of the fee required to break even, as well as both the absolute and relative welfare losses, increases to the point where, for an elasticity of 1.25, the fee is $5.85 per trip, the absolute loss $250,000,000, and the relative loss 11.6%. For the more realistic assumption of an elasticity of .75, the absolute loss is $100 million and the relative loss 1.4%.

While the absolute welfare loss is fairly substantial, the important issue here is political viability. It is most unlikely that any government would be able to get the electorate to support fees of almost $3.00 for a trip downtown in order to make the expressway and subway system self-financing. With fees of the magnitude suggested by this model, it is quite unlikely that urban transportation systems will ever be self-financing, but will forever require subsidies from general tax revenues.

The final aspect of these results that is of interest is the traffic level under various cases, and the capacity that is required to serve it. The capacity figures are shown in Tables 9 to 12, and Table 14 shows the traffic levels for various pricing policies for the medium elasticity case. The model generally builds up expressway capacity faster than subway capacity because its shares and demand functions reflect a common preference for private over public transit. For example, Table 14 shows that, even though a break-even policy discourages a lot of traffic, the traffic that remains is heavily concentrated on the expressway, particularly in the peak. This means that substantial

Table 14. Influence of Pricing on Traffic Levels

(Table entries are passengers in year 40.)
Results are for demand elasticity of -.75, cost parameter of .4

Pricing Policy	Facility and Period				
	Expressway Peak	Expressway Off-Peak	Subway Peak	Subway Off-Peak	Total
Marginal Cost Pricing	16,200 (15%)	35,200 (32%)	12,400 (11%)	44,600 (42%)	108,400 (100%)
Existing Pricing (Subway Fee = 60¢)	23,200 (25%)	37,800 (40%)	10,300 (11%)	23,300 (24%)	94,600 (100%)
Free Subway and Expressway	18,000 (16%)	32,200 (29%)	15,600 (14%)	45,100 (41%)	110,900 (100%)
60¢ peak period fees, free off-peak	15,100 (13%)	36,200 (32%)	10,700 (10%)	50,300 (45%)	112,300 (100%)
Break-even policy ($2.86 fee for both Subway and Expressway)	20,600 (34%)	22,200 (36%)	10,700 (17%)	7,800 (13%)	61,300 (100%)

Note: Percentages of totals for a given pricing policy are listed below table entries.

expressway capacity must be provided. Marginal cost pricing shifts a great deal of traffic to the subway, so as to balance congestion between it and the expressway. The policy of charging a 60¢ peak period fee produces a traffic profile which most closely approximates that of marginal cost pricing, while the existing policy concentrates traffic on the expressway. The model shows that the balance of traffic is quite sensitive to relative prices, and can easily be shifted from one mode or one time to another. This result is highly significant for transportation planners, as it indicates that pricing policy could be used to shift demand out of the urban expressway peak, which will reduce the need for additional expressway capacity.

All in all, this model appears to be quite consistent with the simpler model, both in the magnitude of the results, and in their relative welfare significance. For expressways, like airports, there is a range surrounding the marginal cost pricing, social opportunity cost investment optimum, where the welfare surface is relatively flat. It is probably encompassed by interest rates used for capacity expansion of between 5 and 15 percent, and by user fees anywhere from zero to perhaps $2 per trip. However, as discussed previously, it is unlikely that such high user fees are politically feasible, with the result that the range of politically feasible user fees will always produce a deficit for the urban transportation system.

Finally, the traffic balance among facilities and times seems relatively sensitive to pricing policies, so that prices can be used to shift traffic, which could alleviate the need for additional expressway capacity simply to meet peak period demands.

CHAPTER FIVE
CONCLUSION

With this report, the line of research begun in previous reports has come to its completion. Realistic models of competing transportation facilities have been formulated, and the welfare significance of various pricing and investment policies has been examined. The major gap in previous reports, the lack of substitutability between modes or times of day, has been filled.

The overall result is that for a wide range of policies, the relative welfare losses due to sub-optimal investment and pricing policies are minimal, and the absolute losses, while not negligible, are acceptable. This range can be defined as encompassing interest rates used to decide on investment timing of between 5 and 15 percent (assuming that the optimum is 10 percent) and pricing policies that vary from free facilities (which is simply pure average cost pricing) through existing policies to a level probably double existing policies, both for airports and for urban transportation systems. The difference between airports and urban transportation systems is that pricing policies which will allow the facility to break even are more likely to be politically acceptable in the former case than the latter.

The policy issue facing transportation ministries, then, is what pricing and investment policies to use. If government wants the transportation system to be as close to self-financing as is politically feasible and if it wants to minimize the capital outlays going to transportation facilities, the best approach to take would be to use higher interest rates in determining when to expand facilities (such as 15% rather than 10%) and to implement peak-load pricing, or something approaching peak-load pricing, wherever possible.

The latter results in a better balance of traffic between facilities and times of day, with the result that less capacity is required. This is particularly the case in the urban transportation system, where peak-load pricing reduces required capacity for the expressway.

This study has produced a relatively clear result. The technology is available to implement peak-load pricing in transportation systems. Perhaps its time has finally come.

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APPENDIX A

Transpn Res.-B, Vol. 16B, No. 1, pp. 17–29, 1982
Printed in Great Britain

0191-2615/82/010017-13$03.00/0
Pergamon Press Ltd.

THE EFFECTS OF NON-OPTIMAL PRICING AND INVESTMENT POLICIES FOR TRANSPORTATION FACILITIES

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(*Received* 20 *May* 1980; *in revised form* 15 *December* 1980)

Abstract—A simulation model was used to show the impacts of non-optimal pricing and investment timing policies for a major airport and an urban expressway. It was found that: the losses of economic surplus due to non-optimal pricing and investment policies were relatively small (less than 1% of the net present value of economic surplus for optimal pricing and investment), that marginal cost pricing covers the capital cost of all but the first increment of capacity, that higher user fees will permit the facilities to break even with relatively small losses of economic surplus, that marginal cost pricing has some effect relative to existing pricing policies in delaying the dates new capacity is required and achieving better capacity utilization, and that the shape of the cost function has a substantial impact on the amount of capacity required.

INTRODUCTION

The purpose of this paper is to examine the implications for economic welfare, financial self-sufficiency, and peak and off-peak capacity utilization on non-optimal pricing and investment policies for transportation facilities. In the analytical framework used here, economic welfare is defined as economic surplus, which is the difference between what users would be willing to pay for transportation services and the social cost of providing those services, measured as the difference between the area underneath the demand curve for a transportation service and the area underneath the marginal social cost curve of the service. Non-optimal pricing policies are those where price does not equal marginal cost and non-optimal investment occurs if capacity is added either before or after its incremental benefits equal its incremental costs (Borins, 1981; De Salvo and Lave, 1968).

It is recognized that the use of economic surplus has been criticized both within and outside the economics profession. The two major points of criticism have been that economic surplus should be measured under a demand curve for which real income is held constant (that is, an income-compensated demand curve) and that the addition of surplus across individuals assumes that all have the same marginal utility of income (see Currie *et al.*, 1971, for a discussion of these criticisms). These two criticisms should not present great difficulties for the model in this paper. Because the transportation expenditures modelled here are a small portion of income, income-compensated and ordinary demand curves are almost identical. The policies modelled here have such a small impact on passengers' incomes that the marginal utility of income for all passengers can be assumed to be relatively equal over that range. Because there may be readers who, despite these disclaimers, are disquieted by the use of economic surplus, the paper examines the impact of pricing and investment on two other evaluation criteria: the financial performance, including self-sufficiency, of the investment, and the balance in capacity utilization between peak and off-peak periods. The paper begins by presenting a model used to simulate the growth of demand and capacity at a transportation facility. The model is specified by means of parameters representative of a major international airport and an urban expressway. Simulation results are used to examine the effects of various pricing and investment policies on the three criteria mentioned above: economic surplus, financial performance, and capacity utilization during peak and off-peak periods.

THE MODEL

The model uses a logarithmic demand function of the form:

$$Q_t = A_0(1 + G)^t(F + P)^{-b} \tag{1}$$

†The financial support of Transport Canada is gratefully acknowledged. The views presented here are solely those of the author.

where Q_t is trips per period using the transportation facility in year t, A_0 is a constant term for the first year of the facility's life, G is the annual rate at which the constant term increases, F represents the portion of total time and out-of-pocket trip cost which is independent of congestion at the transportation facility, P represents costs of using the facility, and B is the full-price elasticity of demand for trips. As time passes, the demand curve for trips shifts to the right.

It is recognized that the logarithmic is not the only possible functional form for a demand model. However, the logarithmic form has long been used successfully in transportation demand modelling (Taneja, 1978; Quandt, 1970; Gaudry and Wills, 1978). The multiplicative gravity model, one of the intuitively most appealing demand models, is linear in the logarithms. The logarithmic function specified above has a strong advantage for this study: its constant price elasticity facilitates the sensitivity analysis of demand elasticities. The particular function used here assigns to the constant term the impact of income and prices of goods other than the transportation modes included in the model.

It is assumed that the cross-elasticity of demand between peak and off-peak period trips is zero, which is convenient because the assumption permits the measurement of the area under peak period demand curves which, at any given point in time, do not shift in response to changes in the price of off-peak period trips, and conversely. This assumption should not be a serious distortion of reality because a good deal of research indicates that cross-elasticities across time periods are low for many types of trips, especially business (see Kraft and Domencich, 1972 and Borins, 1978). Finally, to test the sensitivity of the model to changes in functional form, a linear demand function with entirely different parameters was also used, and produced the same results as the model presented here.

The cost of using the facility (P) will depend both on congestion at the facility and on the government's pricing policy. The common form of pricing policy in effect today involves some user fee which is generally constant at all times throughout the life of the facility. In this case, the cost of using the facility is:

$$P = \left[P_o + Z\left(\frac{C}{C - Q_t}\right)^x\right] \tag{2}$$

where P is the cost to the average user of using the facility, P_o is the user fee, Z is a constant equal to the time and operating cost to the average user of using the facility when it is completely uncongested, C is the capacity of the facility, and x is a parameter of the congestion cost function.

When the facility is completely uncongested, Q_t is very small, and the expression $C/(C - Q_t)$ is approximately equal to 1. As the facility gets more crowded, $C/(C - Q_t)$ increases, thus raising the cost of using the facility. One condition of the model is that $Q_t < C$, which allows all users to be serviced by the end of the period. For the case where $x = 1$, the average cost function is that which is derived from steady-state queuing theory (DeSalvo and Lave, 1968). This cost function has been found to accurately represent delays experienced by aircraft (Federal Aviation Administration, 1969; Federal Aviation Administration, 1975; Horonjeff, 1962). For urban expressways estimated volume-delay curves (Keeler and Small, 1977 and Boardman and Lave, 1977) are differently shaped, with traffic moving at close to the intended speed until volume is near design capacity. This can be approximated by using a value of x less than 1. Figure 1 shows the shape of this function for a number of values of x. A value of $x = 0.2$ seems to most closely approximate estimated cost functions. If it were found that time and operating costs increased in different ways as the traffic volume approached capacity the equation could be modified to use a different Z and x for each.

In order to determine the equilibrium number of trips using the facility under existing policies, eqn (2) would be substituted in eqn (1), which, when all the parameters are specified, is a non-linear equation in one unknown, Q_t. In order to maximize economic surplus, transportation facilities should be priced at marginal cost. Marginal cost can be found by multiplying the average cost of making a trip in real resources† by Q_t to yield total cost which is

†This excludes P_o, since P_o is an arbitrary user fee, not a real resource cost.

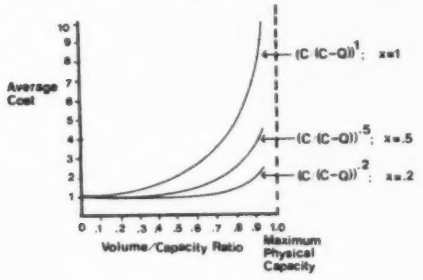

Fig. 1. A family of average cost curves.

$$Q_t\left[F + Z\left(\frac{C}{C - Q_t}\right)^x\right]. \tag{3}$$

Taking its derivative with respect to Q_t then yields

$$F + Z\left(\frac{C}{C - Q_t}\right)^x\left[1 + \frac{xQ_t}{(C - Q_t)}\right]. \tag{4}$$

For the case where $x = 1$ eqn (4) can be reduced to

$$F + Z\left(\frac{C}{C - Q_t}\right)^2. \tag{5}$$

In order to find the optimal level of traffic given marginal cost pricing, eqn (4) would substituted into eqn (1) in the place of $F + P$ and eqn (1) would be solved for Q_t. In this case P_0 would be equal to zero. Because the price perceived by the user of a transportation facility is always equal to the average cost of using the facility, to implement marginal cost pricing it is necessary to charge a toll equal to the difference between marginal and average cost of using the facility, which would be

$$Z\left(\frac{C}{C - Q_t}\right)^x\left[1 + \frac{xQ_t}{C - Q_t}\right] - Z\left(\frac{C}{C - Q_t}\right)^x \tag{6}$$

which equals

$$Z\left(\frac{C}{C - Q_t}\right)^x\left[\frac{xQ_t}{(C - Q_t)}\right] \tag{7}$$

which is zero when Q_t is zero, and increases as Q_t increases. The difference between this toll and P_0 is that, unlike P_0, this toll would not be fixed over the life of the facility.

The economic benefit of capacity expansion in any given period is equal to the economic surplus resulting from a larger facility with a capacity of $C + K$ units minus the economic surplus from a smaller facility with a capacity of C units. The increase in economic surplus (ΔS) due to capacity expansion at time t can be written as

$$\left(\int_0^{Q_{t,C+K}} P(Q)\mathrm{d}Q - \int_0^{Q_{t,C+K}} MC_{C+K}\mathrm{d}Q\right) - \left(\int_0^{Q_{t,C}} P(Q)\mathrm{d}Q - \int_0^{Q_{t,C}} MC_C\mathrm{d}Q\right) \tag{8}$$

where the subscripts $C + K$ and C indicate the capacity.†

†If the demand curves were linear, ΔS could be approximated by the procedure outlined in Turvey (1974)

In order for this formulation to be valid, it is assumed that there are no distortions anywhere else in the economy so that no problems are raised by the theorem of the second best. To incorporate peak and off-peak periods, a different demand function was specified for each. In order to evaluate the integrals, their lower bounds were set at 1, because the inverse demand function is asymptotic to the vertical axis. Using this lower bound will result in a very slight understatement of the economic surplus associated with any facility, but will not affect the measured increase in economic surplus due to additional capacity.

The benefits of additional capacity must be compared with the costs, in order to determine when to add capacity. There are two costs associated with additional capacity; the capital cost, which is incurred in the year the facility is brought on line, and an annual operating cost which is a function of the size of the facility but which is independent of the traffic using the facility. Capacity should be expanded when

$$\Delta S \geq r.K + M \tag{9}$$

where r is the social discount rate, K is the capital cost, and M the operating cost (see Harberger, 1974 and Marglin, 1963). Since this model uses lumpy rather than infinitely divisible transportation facilities, expansion would occur in the first year when ΔS associated with a given unit of additional capacity equals or exceeds $rK + M$. The choice of a social discount rate to represent the value of investing these funds elsewhere in the economy has been subject to extended conceptual and empirical debate among economists. This model simply assumes that it is possible to define a social discount rate. Therefore, non-optimal capacity expansion is defined to occur when the facility planners, in determining expansion dates, use a value of r which is either greater or less than the "real" social discount rate.†

Finally, the net present value of economic surplus less capital cost of any pricing and investment policy over a planning horizon, of T years is equal to:

$$\sum_{t=0}^{T} \frac{1}{(1+r)^t}\left[\int_0^{Q_t} P(Q_t)dQ_t - \int_0^{Q_t} MC\,dQ_t - M_t - K_t\right] \tag{10}$$

It should be noted that K is equal to zero except in those years when new capacity is added.

SPECIFICATION OF THE MODEL

The model was run for a 40-year planning horizon, during which time demand grew and new facilities were added. In each year, the model was solved by means of the Newton-Raphson method for both existing and possible expanded capacity, in order to determine when to expand capacity. The model kept track of the net present value of economic surplus, as well as marginal cost pricing fees and the net present value of financial cost and revenue flows in every year.

The model was specified for both an airport and an urban expressway. The parameters were based on data for Toronto International Airport and data on Ontario highways, available from the Ontario Ministry of Transportation and Communications. Complete documentation can be found in Borins (1980) which is available from the author.

The airport model represents an integrated facility providing one runway which can accommodate 40 aircraft operations per hour and sufficient terminal and access road capacity to handle the resulting passenger flow. The airport can be expanded by adding similar runway-terminal-access road units. The expressway model represents an urban expressway 16 km. in length, which is expanded one lane at a time in both directions. Key parameters for both the demand and capacity sides of the model are presented in Table 1. In order to provide a sensitivity analysis of variations in the value of passengers' time and demand elasticity, high

†Wheaton (1978) developed a model where non-optimal expansion could occur because planners overestimated ΔS. Wheaton's model clearly shows how ΔS could be overestimated when the increment to capacity is small, but it is unclear how this overestimate of ΔS would be operationalized when the increment to capacity is large, which is the empirically relevant case (Borins, 1981). The procedure used here, as will be shown below, permits substantial over-and under-estimates of capital cost, and errors in the timing of investment which are larger than Wheaton presents.

Table 1. Data for pricing and investment models

	Airport model	Urban expressway model
Capacity of basic unit	airport to handle 40 aircraft/hour	16 km. two-lane expressway; 2,000 autos/lane/hour
Land cost	$100,000,000	$ 90,000,000
Capital cost of initial capacity	$800,000,000	$ 7,500,000
Capital cost per additional unit of capacity	$800,000,000	$ 4,250,000
Annual operating and maintenance cost per basic unit	$ 30,000,000	$ 90,000
Annual rate of growth of demand	5%	5%
Initial demand level		
- peak period	10 aircraft/hour	500 autos/lane/hour
- off-peak period	5 aircraft/hour	225 autos/lane/hour
Full price elasticity of demand		
- peak period	.75, 1.25	.75
- off-peak period	1.25, 2.00	1.50
Value of passengers' time per hour		
- peak period	$ 6.50, $11.50	$3.00
- off-peak period	$ 6.50, $11.50	$ .55
Component of full price independent of congestion (F), per passenger		
- peak period	$193.00, $218.00	$2.10
- off-peak period	$193.00, $218.00	$1.30
Load factor		
- peak period	65% of aircraft seats	1.5 people/car
- off-peak period	65% of aircraft seats	1.7 people/car
Existing user fees, peak and off-peak periods	$1,100 per flight	gasoline tax of $.04¢/litre

Source: Borins (1980)

and low estimates of those two parameters are presented. Finally, the model uses a social discount rate of 10% to compare constant dollar flows, which is based on Jenkins' (1972) estimate of the rates of return on private and public sector investments in Canada.

THE EFFECTS OF NON-OPTIMAL PRICING AND INVESTMENT ON ECONOMIC SURPLUS

The airport model, for which the parameter $x = 1$, provides a simple measure of the extent of deviations of pricing policy from marginal cost. When $x = 1$, compare eqns (2) and (5). For average cost pricing, the exponent of $C/(C - Q_t)$ is 1, while for marginal cost pricing it is 2. Thus there is a continuum of exponents e where $e < 1$ indicates pricing at less than average cost, $1 < e < 2$ pricing between average and marginal cost, and $e > 2$ pricing above marginal cost.

Table 2 indicates the net present value of economic surplus for the airport model for a variety of pricing policies as well as high and low estimates of the value of passengers' time and the elasticity of demand. Table 3 shows the net present value of economic surplus for the expressway model for marginal cost and existing pricing policies and for a number of different values of x, the parameter defining the shape of the average cost curve. In all cases, the losses of economic surplus due to non-marginal cost pricing are a very small percentage, almost always less than one percent, of the optimal level of economic surplus associated with marginal cost pricing. This is consistent with a large body of economic research which has found equally small relative losses of economic surplus due to the non-optimal pricing of transportation facilities (Kraus *et al.*, 1976), to the high prices charged in monopolistic or oligopolistic markets (Harberger, 1954; Schwartzman, 1960), or to tariff barriers (Janssen, 1961; Johnson, 1958; Wemelsfelder, 1960; Scitovsky, 1958).

In absolute terms, the losses due to non-optimal pricing of the airport are significant. For example, the loss of net present value due to present pricing policies varies between $11 and $39 million dollars. The magnitude of this loss should be more than enough to justify the implementation and enforcement costs of a pricing policy which more closely approximates the

Table 2. Effects of pricing policy on economic surplus, airport model

(Unless otherwise indicated, table entries are $ x 10^6)

Pricing policy	Assumptions about demand function											
	High time value, low elasticity			High time value, high elasticity			Low time value, low elasticity			Low time value, high elasticity		
Pricing policy Exponent e =	surplus	$loss	%loss	surplus	$loss	%loss	surplus	$loss	%loss	surplus	$loss	%loss
.25 (P<AC)	46935	22	.05	9828	38	.4	41064	19	.05	8502	49	.57
1.00 (P = AC)	46946	12	.02	9847	19	.2	41073	10	.02	8518	33	.38
1.50 (AC<P<MC)	46953	4	.01	9861	5	.05	41080	3	.01	8546	5	.06
2.00 (P = MC)	46957	0	0	9866	0	0	41083	0	0	8551	0	0
3.00 (P>MC)	46940	13	.04	9847	19	.2	41067	16	.04	8501	50	.58
4.00 (P>MC)	46894	63	.10	9798	68	.7	41029	54	.13	8479	72	.84
e = 1; P_0 =												
0	46946	12	.02	--	--	--	41073	10	.02	8518	33	.38
$ 400	46950	7	.01	--	--	--	41075	8	.02	8523	28	.33
$ 1,100	46946	11	.02	--	--	--	41068	15	.04	8512	39	.46
$ 4,000	46805	152	.32	--	--	--	40920	163	.40	8263	288	3.37
$ 10,000	46141	817	1.74	--	--	--	40173	910	2.21	7189	1360	15.93

-- results not calculated

Table 3. Effects of pricing policy on economic surplus, expressway model

(Table entries are $ x 10^3)

x (shape of average cost curve)	Marginal cost pricing	Existing pricing surplus	deviation $	deviation %
.2	1,360,378.8	1,360,215.3	163.5	.01
.4	1,357,814.6	1,357,677.4	137.2	.01
.6	1,355,835.4	1,355,703.3	132.1	.01
.8	1,354,077.2	1,353,932.0	145.2	.01
1.0	1,352,353.7	1,352,193.8	160.0	.01

marginal cost optimum. On the other hand, the deviation from the optimal level of economic surplus due to existing expressway pricing policies is quite small, approx. $150,000 in net present value. It is most unlikely that this loss (which is equivalent to a continuous loss of $15,000 per annum, at a 10% discount rate) could justify the implementation and enforcement costs of a marginal cost pricing scheme for the expressway.

A number of other aspects of the airport model are noteworthy. The level of economic surplus is higher for low demand elasticities than for high demand elasticities because in the latter case the demand function approaches the vertical axis more rapidly. Furthermore, the relative deviations from the maximum economic surplus are greater for high than for low demand elasticities because for the former the traffic level is affected more substantially by pricing policy. The assumption made about the value of passengers' time does not affect the outcome greatly, either for the relative or absolute losses due to non-marginal cost pricing.

The lower section of Table 2 shows that when very heavy user fees ($4000 and $10,000 per flight) are imposed, there are substantial losses of economic surplus relative to the marginal cost pricing optimum, especially when the elasticity of demand is high. For all three assumptions about passengers' time values and demand elasticities shown in the table, the value of economic surplus is higher when user fees are $400 per flight than when they are either zero (a pure average cost pricing policy) or when they are $1100 (the existing pricing policy). This result can be explained by maximizing economic surplus with respect to a value of P_0 which is held constant over the life of the facility.

The net present value of economic surplus for a facility (S) can be rewritten as

$$\sum_{t=0}^{T} \frac{1}{(1+r)^t} \left[\int_0^{Q_t} Q(P_t)dQ - AC_tQ_t \right]. \tag{11}$$

In this formulation, the subscripts indicate the year, total cost is written as $AC.Q$ rather than the integral of the marginal cost curve, and capital and operating and maintenance costs are ignored, since they are not a function of P_0. Also the price, P_t, implicitly includes F, the costs not associated with the facility. Taking the derivative with respect to P_0,

$$\frac{\partial S}{\partial P_0} = \sum_{t=0}^{T} \frac{1}{(1+r)^t} \left[P_t \frac{\partial Q_t}{\partial P_t} \cdot \frac{\partial P_t}{\partial P_0} - MC_t \frac{\partial Q_t}{\partial P_t} \cdot \frac{\partial P_t}{\partial P_0} \right]. \tag{12}$$

Since

$$\frac{\partial P_t}{\partial P_0} = 1 \text{ and } \frac{\partial Q_t}{\partial P_t} = -BA_0(1+G)^t(P_t)^{-B-1}$$

$$\frac{\partial S}{\partial P_0} = \sum_{t=0}^{T} \frac{1}{(1+r)^t} \left[-BA_0(1+G)^t(P_t)^{-B-1}(P_t - MC_t) \right]. \tag{13}$$

Setting $(\partial S/\partial P_0) = 0$, and recalling that $P_t = P_0 + AC_t$,

$$0 = \sum_{t=0}^{T} \frac{(1+G)^t}{(1+r)^t} \frac{(P_0 + AC_t - MC_t)}{(P_0 + AC_t)^{B+1}}. \tag{14}$$

In expression (14), all terms in the denominator are greater than zero. In order that the entire expression equal zero, at least one term in the numerator must be less than zero. Therefore, $P_0 + AC_t - MC_t < 0$ for at least one value of t. Since demand is increasing over time and P_0 is constant, $P_0 + AC_t - MC_t > 0$ (that is, price is greater than marginal cost) in the early years of the facility and $P_0 + AC_t - MC_t < 0$ (price is less than marginal cost) in the later years of the facility. More detailed calculations using the model, not presented here, showed that economic surplus was maximized in most cases, when P_0 was approximately equal to \$500.

The model was also used to calculate congestion tolls which would be charged if marginal cost pricing were in effect. A sample of such tolls is presented in Table 4. In both the airport and expressway cases, the tolls are higher during peak than the off-peak period, and increase substantially as the facility reaches capacity. The relationship between peak and off-peak tolls is similar to that calculated by Kraus *et al.* (1976, Tables 1 and 2) and by Keeler and Small (1977, Table 5). There is a major difference between this model and the others. Both other models assume infinitely divisible capacity, which implies that the facility is always at optimal capacity, that capacity is being expanded continuously, and that the peak and off-peak tolls never change. Because the model in this paper makes the more realistic assumption of lumpy capacity, both peak and off-peak tolls increase over the life of the facility.

The model was also used to investigate the magnitude of losses in economic surplus due to non-optimal investment timing. A wide range of interest rates, from 0.5 to 50%, was used to determine when to expand capacity. The net present values of economic surplus were determined in each case, using as a discount rate the assumed "correct" social discount rate of 10%. To my knowledge this procedure has not been used elsewhere in the economics literature.

Table 4. Sample congestion tolls

	Peak period	Off-peak period
1. Airport model, initial capacity of 40 aircraft/hour, low time value, high elasticity		
Year 0 (initial year)	\$ 140/aircraft	\$ 50/aircraft
Year 29 (last year before expansion)	\$3970/aircraft	\$570/aircraft
2. Expressway model, α=.2, initial capacity of 2 lanes		
Year 0 (initial year)	\$ .08/auto	\$ .02/auto
Year 21 (last year before expansion)	\$ .45/auto	\$ .06/auto

Table 5. Effects of non-optimal investment timing, airport model

Interest rate used for expansion decision	Low time value, high elasticity						High time value, low elasticity					
	Marginal cost pricing			Existing pricing			Marginal cost pricing			Existing pricing		
	surplus	$loss	%loss	surplus	$loss	%loss	surplus	$loss	%loss	surplus	$loss	%loss
.5%	8494.5	56.1	.66	8484.8	65.9	.77	46888.6	68.9	.15	46879.2	78.3	.17
5.0%	8531.5	19.2	.22	8506.8	43.9	.51	46950.8	6.6	.01	46941.2	16.3	.03
10.0%	8550.7	0	0	8511.5	39.2	.46	46957.5	0	0	46946.2	11.3	.02
15.0%	8547.2	3.4	.04	8524.3	26.4	.31	46951.8	12.9	.03	46943.2	14.3	.03
50.0%	8508.8	41.9	.49	8483.5	67.2	.79	46908.8	48.7	.10	46918.6	38.9	.08

Note: In all cases, the social discount rate was 10%. Thus, the net present values of economic surplus were calculated with a 10% discount rate.

A number of economists with whom I discussed this idea while the research was in progress expected that non-optimal investment timing would result in substantial deviations from the optimal level of economic surplus. The actual results are presented in Table 5.

The relative losses in economic surplus, even for interest rates such as 0.5 and 50%, are very small, less than 1%. This counter-intuitive result occurs because neither excess capacity due to low interest rates nor insufficient capacity due to high interest rates affect the overall number of trips very much. One reason for this is that the demand functions have as their argument the full price of trip-making, which includes substantial costs which are independent of facility pricing (see Table 1) and which consequently reduce the elasticity of the demand function with respect to facility pricing and investment policies. As is consistent with the pricing policy results, the more elastic demand function causes greater percentage losses in economic surplus than the less elastic demand function, because the effect of capacity on demand, as mediated through prices, is greater in the former case.

If it is confirmed in other studies, the result that non-optimal investment timing policies have little effect on the level of economic surplus has an important implication for economists. While economists argue about the value of the "correct" social discount rate, all would agree that it lies somewhere between 5 and 15%. This model shows that the impact of incorrect discount rates between 5 and 15% on the level of economic surplus is almost negligible. This suggests that, at least in the area of transportation investment timing, economists have allocated too many resources to determining the "correct" social discount rate because the cost of not knowing the "correct" rate, if it exists, is not great.

One final result indicated in Table 5 is that the deviations from the optimal value of economic surplus due to both non-optimal pricing and non-optimal investment timing are roughly equal to the sum of the deviations due to each factor separately. This can be shown for the "high time value, high elasticity" case. The deviation due to existing pricing for the optimal 10% social discount rate is $11.3 million. For an interest rate of 5%, the deviation when there is marginal cost pricing is $6.6 million. The deviation for an interest rate of 5% and existing pricing is $16.3 million, which is approximately equal to $11.3 plus $6.6 million. This result does not hold so well in every case. Because the investments are lumpy, and because net present values were only calculated for 40 years, there are some cases where net present values of economic surplus are higher for non-optimal interest rates than for the optimal 10% rate or higher for existing pricing than for marginal cost pricing. Over an infinite planning horizon, the expected result is that economic surplus would be higher for optimal pricing and investment policies than for non-optimal ones.

In summary, the major finding of this section is that there is a wide range of non-optimal pricing and investment timing policies over which the losses in economic surplus are relatively small. Put differently, for a wide range of values around the optimal pricing and investment timing policies, the economic surplus surface is quite flat. The implication of this result, if it is confirmed in other studies, is that the maximization of economic surplus might not be the primary objective of transportation planners, because the cost of not achieving it does not appear to be great. Furthermore, this suggests that it would be worthwhile to investigate the impacts of pricing and investment policies on alternative criteria for evaluation, such as financial performance and balanced capacity utilization. These criteria are discussed in the two following sections.

THE EFFECTS OF NON-OPTIMAL PRICING AND INVESTMENT POLICIES ON FINANCIAL PERFORMANCE

The financial implications of various pricing and investment policies were examined by computing the net present value of revenues (user fees minus capital and operating and maintenance costs). Starrett (1978) demonstrated theoretically that for recursive lumpy investments (such as those in this paper) marginal cost pricing should be sufficient to cover all capital requirements *except* the initial investment. Table 6 presents the net present value of revenues for both the airport and expressway models. For the airport model, only the low time value, high elasticity scenario is shown, because the other scenarios led to similar results. In the airport model, the loss of $935,000,000 for marginal cost pricing is slightly greater than the $900,000,000 cost of the original facility. For the expressway model, the loss of $85,400,000 for marginal cost pricing is less than the $97,500,000 cost of the initial facility. Both results are roughly consistent with Starrett's theoretical demonstration. For the airport model, the existing pricing policy results in a loss of $567 million, which is 60% of the initial capital cost: for the expressway model, present gasoline taxes do not even cover the $7.5 million cost of the first two lanes, let alone the land acquisition cost of $90 million. However, a $2.00 expressway toll does cover about half the initial investment. In practice, the road financing picture is not as bleak as is painted by this model, since the gasoline tax is assessed on all travel, and local trips on existing roads cross-subsidize the construction of new expressways.

Another question explored by the airport model is that of the level of user fees which would be necessary to permit the airport to break even. In this case, it is assumed that such fees would be assessed equally on all traffic, rather than differentiated by time of day. Two alternative definitions of breaking even were considered. The first was a policy requiring every basic unit of capacity to be self-sufficient by the time the next unit was begun. The second was a policy requiring a number of consecutive units to be self-sufficient collectively, for example, the first two units by the time the third is begun. The first policy allows the user fee to change whenever a new unit is begun, while the second constrains it to be constant over the units treated collectively.

Table 7 shows some results for those two policies. In order for the first unit to break even

Table 6. Effects of pricing policies on financial self-sufficiency

(Table entries are $ x 10^6)

Pricing policy	Net present value of revenues
1. Airport model: low time value, high elasticity	
Average cost	- 1323.8
Marginal cost	- 935.6
Existing (P_0 = $1,000)	- 566.9
2. Expressway model: x = .2	
Average cost (no gasoline tax)	- 98.4
Existing (gasoline tax = $.046/litre)	- 91.2
Marginal cost pricing	- 85.4
$2.00 expressway fee, no gasoline tax	- 49.5
$2.00 expressway fee, $.046/litre gasoline tax	- 47.4

Note. In both cases, capacity expansion decisions were based on a 10% interest rate, and net present values were calculated for a 40 year period, using a 10% discount rate.

Table 7. Values of user fee that allow airport to break even financially

Scenario	Different fee for each unit of capacity		Constant fee for the first and second units of capacity		
	first unit	second unit	user fee	loss of surplus $ x 10^6	%
high time value, low elasticity	$2130	$1070	$1970	35.0	.07
low time value, high elasticity	$2230	$ 930	$2080	72.8	.85

Note: a 10% interest rate was used for capacity expansion decisions.

on its own, user fees must be over $2000 per aircraft operation, which is almost twice the present level. However, if the second unit alone is to break even, the user fees need only be $1000 per aircraft operation, which is less than the current policy. The reason the required fees drop sharply for the second unit alone is that fees are collected from all users of the facility, whether they use the first or the second unit. If fees are set to cover the first and second units together, they are slightly lower than fees to cover the first unit by itself, somewhat less than $2000 per aircraft operation. They are no lower because discounting reduces the net present value of revenues and expenditures during the later years of the planning horizon, so that revenues must be gained in the early years to cover the capital cost of the first unit. The implications for economic surplus of the constant fee policy are also presented. The deviations are less than 1% relatively, but substantial in dollar terms. It is possible to calculate the loss of surplus by moving from the existing policy to a policy designed to break even. Assuming the high value of passengers' time and low elasticity of demand, net revenues would increase by $564.4 million (from a loss of $564.4 million to 0) while economic surplus would be decreased by $35,000,000–$11,000,000 = $24,000,000. Under the low time value and high elasticity assumption, increased revenues of $566 million would be had at the cost of an increased welfare loss of $50,000,000 ($72.6 minus $22.9 million). In both cases, the increased revenue is much larger than the loss in economic surplus. This occurs because the full price demand functions are relatively inelastic to changes in the price of using the facility.

These results have interesting implications for managers of transportation facilities. In 1969, the Canadian Ministry of Transport set up an Airports Revolving Fund in order to implement the goal of making major airports financially self-sufficient. In establishing the fund, no capital charges were assessed for airports then in existence: capital charges were assessed only for new airports. Also, each airport's accounts were kept separately. For example, Toronto International Airport was not required to cover capital costs for its initial basic unit of capacity. As would be expected from this model, it has been able to cover operating and maintenance costs as well as capital charges for a second unit of capacity. On the other hand, the new Montreal International Airport (Mirabel) was required to cover its initial capital charges. This model suggests that, given existing user fees, Mirabel will always operate at a loss. The argument advanced above the existing units of capacity can be used to finance new units of capacity could be generalized to apply to the airports in the Airports Revolving Fund. If they are seen as a transportation network of inter-related airports, then it would be rational to use surplus revenues from existing airports (Toronto, Montreal Dorval, Vancouver) to finance additions to the capacity of the network, such as Mirabel or the expansion of Calgary International Airport.

THE EFFECTS OF NON-OPTIMAL PRICING AND INVESTMENT ON CAPACITY UTILIZATION

The effects of non-optimal pricing and investment policies on the timing of investments, on the net present value of costs, and on capacity utilization were also investigated. Table 8 shows the effects on investment timing and costs for the airport model, given the low time value and high demand elasticity scenario.

Greatly different interest rates used to determine when to expand capacity significantly

Table 8. Effects of pricing policies and interest rates used for capacity expansion decision on timing of investments and total costs, airport model with low time value and high elasticity

Pricing policy	Interest rate used (%)	Year investment undertaken: second unit	third unit	Net present value of costs $ \times 10^6$
Existing pricing	.5%	23	35	1377
(P_0 = $1100)	5.0	26	38	1335
	10.0	28	40	1314
	15.0	30	--	1282
	50.0	35	--	1257
Marginal cost pricing	.5%	22	34	1393
	5.0	27	39	1324
	10.0	30	--	1282
	15.0	32	--	1270
	50.0	39	--	1244

Note: Costs were calculated up to year 40.
-- indicates that third unit was not built before year 40.

affect the opening date: an interest rate of 0.5% advances the opening date by 11–18 years compared to an interest rate of 50%. However, the difference between interest rates of 5 and 15% which bracket the range policy makers are likely to use, is 3–5 years. The net present value of costs does not vary greatly as a function of the interest rate used. The difference between 0.5 and 50% interest rates is about $140 million, while the difference between 5 and 15% is about $50 million. This happens because the main component in the cost stream is the $900 million expenditure in the first year, and differences in opening dates of additional units are made financially less significant by the 10% discount rate.

Marginal cost pricing results in earlier capacity expansion than existing capacity for low interest rates, while the opposite is true for high interest rates. At low interest rates, the airport is expanded at low levels of capacity utilization for which existing user fees are higher than marginal cost pricing tolls, and therefore full prices are lower and demand higher for marginal cost pricing. The converse occurs at high interest rates. The capacity expansion date calculations for the other scenarios, not presented in Table 8, show that the high value of passengers' time advances opening dates by two to three years, *ceteris paribus*, and that low demand elasticities advance opening dates by one to two years, *ceteris paribus*. The former occurs because the cost of delaying expansion is greater if time is more valuable and the latter because increased congestion discourages more traffic when demand elasticity is high than when it is low.

Table 9 compares the effects of the shape of the cost function and of pricing policy on the amount of capacity provided in the expressway model. The table shows that, after 40 years, only two lanes in either direction are needed for low values of x, while high values result in 8 or 9 lanes in either direction. This occurs because the area underneath the cost curve is greater when x is greater, and therefore the increase in economic surplus due to additional capacity will also be greater. The practical implication of this is very important. Transportation planners must devise accurate estimates of the volume-delay curves for a given transportation facility, since the shape of the volume-delay curve determines the shape of the cost curve and thereby influences the amount of capital needed. By contrast the influence of pricing on capacity requirements is very limited. In all cases, the ultimate capacity is the same regardless of the pricing policy. More detailed examination of the results, not presented in the table, showed that for $x = 0.2$, both the existing and marginal cost pricing policies led to the addition of a second lane in each direction in year 23. Even the $2.00 per car expressway toll delayed the second lane only to year 28.

Table 10 examines capacity utilization during peak and non-peak periods. The table shows

Table 9. The effect of the shape of the average cost curve on capacity expansion, expressway model

Table entries are ultimate capacity of expressway (lanes in either direction)

	Value of x (shape of average cost curve - see fig. 1)				
	.2	.4	.6	.8	1.0
Pricing policy					
Marginal cost	2	4	5	6	8
Existing policy	2	4	5	6	8

Table 10. The effect of pricing policy and interest rates used for capacity expansion decisions on airport capacity utilization (given low time value, high elasticity)

Pricing policy	Interest rate (%) for capacity expansion decision	Aircraft/hour prior to expansion year, initial capacity unit		
		peak	non-peak	non-peak/peak
Existing	.5%	28.6	14.3	.500
	5	29.5	15.0	.508
	10	31.8	16.5	.519
	15	33.9	19.1	.534
	50	37.6	23.9	.636
Marginal cost pricing	.5%	24.5	13.4	.547
	5	28.3	16.7	.590
	10	30.0	19.0	.630
	15	31.0	20.6	.665
	50	33.3	25.9	.778

capacity utilization in the year prior to expansion, which is the highest capacity utilization which would occur in the life of a given unit. Marginal cost pricing produces better capacity-utilization than existing pricing, in that there is a more even balance of traffic between peak and non-peak periods. This becomes more pronounced as the interest rate used to decide on capacity expansion is increased. This occurs because a higher interest rate criterion results in heavier traffic in the years before expansion. Congestion prices become much higher in the peak period relative to the non-peak period, and the balance of traffic is shifted towards the non-peak period. The table shows that marginal cost pricing shifts one plane per hour out of the peak into the non-peak when a 10% interest rate criterion is used, and up to three to four planes per hour when a 50% interest rate criterion is used. It should be noted that this is based on a model which assumed that cross-elasticities of demand between peak and non-peak periods are zero. If the model permitted higher cross-elasticities, there would be more shifting.

CONCLUSION

This paper has developed a simulation model of a transportation facility to examine the impacts of non-optimal pricing and investment policies upon economic surplus, financial performance, and capacity utilization. The model was specified using Canadian data to represent a major airport and an urban expressway. This specification should be broadly representative of such facilities in other countries as well. The following are some of the major results:

(1) The deviations from maximal economic surplus due to non-optimal pricing and investment are relatively small, generally less than 1% of the net present value of economic surplus for optimal pricing and investment. The cost of non-optimal pricing and non-optimal investment are additive, rather than multiplicative. The finding that interest rates used to determine capacity expansion which differ greatly from the "correct" social discount rate have little effect on the level of economic surplus was surprising. If confirmed in future research, these results suggest that the maximization of economic surplus might not be a primary criterion in determining pricing and investment policies for transportation facilities, and that greater attention ought to be paid to other criteria, such as financial self-sufficiency and balanced capacity utilization. In particular, if there is little loss in economic surplus involved in making transportation facilities self-sufficient there may be a strong argument for doing so, provided that the costs do not fall mainly on the poor.

(2) Marginal cost pricing will cover the cost of all capital charges except for the initial capacity. The costs of later units of capacity are easier to cover, since they can be spread over a greater volume of traffic than those of the first unit. The cost of the initial capacity can be covered by raising user fees and departing from marginal cost pricing. For the airport, this would necessitate a doubling in existing user fees. However, the deviation from maximal economic surplus would still be relatively small, less than 1%.

(3) The shape of the cost function has a substantial impact on the amount of capacity required. This makes clear the importance to transportation planners of accurate estimates of volume-delay curves.

(4) Marginal cost pricing, relative to existing pricing policies, has some effect in delaying the dates new capacity is required, and providing for more balanced traffic between peak and non-peak.

The logical extension of this research would be to elaborate both the supply and demand sides of this model, the former by modelling transportation networks and the latter by using more complicated demand functions, in order to determine whether or not the results obtained here hold in those contexts as well.

Acknowledgements—The research assistance of Bladdy Au and the comments of Michael Wills, Don Dewees, and two anonymous referees are greatly appreciated.

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APPENDIX B

```
C*********************************************************************
C
C  THIS IS THE MAIN PROGRAM FOR THE URBAN TRANSIT MODEL.
C  TO FUNCTION THIS PROGRAM MUST BE LINKED TO FILES DECOMP, SOLVE,
C  DERIV, REVV, SRPLS AND CHART.
C
C  VARIABLE DEFINITIONS (ROUGHLY IN ORDER OF APPEARANCE):
C
C  A,B,CC - TRANSLOG DEMAND FUNCTION CONSTANTS
C  Q - QUANTITY VECTOR
C  P - PRICE VECTOR
C  S - SHARES VECTOR
C  Y - SOLUTION VECTOR OBTAINED FROM SOLVE
C  CCS,OCS,CCE,OCE - CAPACITY/OPERATING COSTS FOR SUBWAY/EXPRESSWAY
C  REV - REVENUES VECTOR
C  Z,X1,X2 - CONSTANTS FROM THE COST FUNCTIONS
C  A0, A1 - CONSTANTS FROM OVERALL COST FUNCTION
C  POP - PART OF PRICE INDEPENDENT OF DEMAND
C  P0 - TOLL VECTOR
C  IPVT - PIVOT VECTOR USED IN DECOMP FOR GAUSSIAN ELIMINATION
C  WORK - VECTOR WORK SPACE FOR DECOMP
C  C - CAPACITY VECTOR
C  C0 - INITIAL CAPACITY VECTOR
C  J - THE 4x4 JACOBIAN MATRIX (THE HEART OF THE WHOLE PROGRAM)
C  ICC - INITIAL CAPACITY COST
C  IR - INTREST RATE
C  NPVS - NET PRESENT VALUE OF SURPLUS
C  CINC - CAPACITY INCREMENT ARRAY
C  YEAR - YEAR IN PROGRESS
C  RATE - RATE PER YEAR THAT DEMAND GROWS
C  RR - SOCIAL DISCOUNT RATE
C  PP - PRICE GIVEN BY TRANSLOG FUNCTION
C  QQ - QUANTITY GIVEN BY TRANSLOG FUNCTION
C  TAU - TOLERENCE FOR SOLUTION ACCURACY; ALWAYS USE TAU.LE.0.01
C  OCSUB,OCEX - KEEP TRACK OF PVOC FOR SUBWAY/EXPRESSWAY
C  PVOC - PRESENT VALUE OF OPERATING COSTS
C  PVR - PRESENT VALUE OF REVENUE
C  PVCC - PRESENT VALUE OF CAPITAL COSTS
C  CRTRN - CRITERION USED IN THE DECISION TO EXPAND
C  TEST - TESTS FOR SOLUTION'S ACCURACY
C  SURPLS - AREA UNDER ALL COST CURVES
C  TR - TOTAL REVENUE
C  EXPRSS,SUBWAY - LOGICAL VARIABLES WHICH SAY WHICH FACILITY IS
C                  CURRENTLY EXPANDED
C  AUD - AREA UNDER THE TRANSLOG DEMAND CURVE
C  NS,NE - LEVEL OF EXPANSION COUNTERS
C  SAVE - A VARIABLE TO SAVE THE VALUE OF SURPLUS IN
C  DS - DELTA S OR DELTA W
C  FLAG1,FLAG2 - VARIABLES WHICH TELL WHICH FACILITY IS TO BE
C                EXPANDED
C
C*********************************************************************
      implicit double precision(a-h,o-z)
      LOGICAL EXPRSS/.FALSE./,SUBWAY/.FALSE./,FLAG1/.FALSE./
      LOGICAL FLAG2/.FALSE./
      DIMENSION B(4),CC(4,4),Q(4),P(4),S(4),Y(4),CCS(7),OCS(7)
      DIMENSION CCE(10),OCE(10),REV(4),z(4),p0(4)
      dimension ipvt(4),work(4),c(4),p0(4),C0(4)
      double precision J(4,4),ICC,IR,NPVS,CINC(10,4)
      integer year/0/,ns/1/,ne/1/
      data rate/1.025/,rr/0.1/,IR/0.1/
C  THIS COMMON BLOCK IS USED IN SEVERAL OF THE SUB-PROGRAMS
C
```

```
      common/save/ a,b,cc,s,r,g,PP,p0,p0,c,a0,a1,x1,x2,z
C  AN EXPLANATION OF THE INPUT DATA CAN BE FOUND AT THE END OF R.DAT
      open(unit=4,file='r.dat',status='old')
      open(unit=3,file='out.dat',status='old')
C  READ IN DATA: TAU IS SUPPLIED BY YOU FROM THE TERMINAL
      read(5,8) tau
      read(4,8) a,b,cc
      read(4,8) x1,x2,a,a0,a1
      read(4,8) p0,c0,z
      READ(4,8) CINC
      READ(4,8) CCE,OCE,CCS,OCS
      READ(4,8) ICC
      read(4,8) p0
      do 5 i=1,4
   5  c(i)=c0(i)
C  WRITE OUT THE INITIAL CONDITIONS
      write(3,105)
      write(3,100) a,b,((cc(i,k),k=1,4),i=1,4)
      write(3,101) x1,x2,a,a0,a1,z
      write(3,103) p0,c
      WRITE(3,110) ((CINC(I,K),K=1,4),I=1,15)
      WRITE(3,203) CCE,OCE,CCS,OCS
      WRITE(3,205) ICC
  205 FORMAT('ICC=',F12.0)
  203 FORMAT('CCE=',10F12.0,/'OCE=',10F12.0,/'CCS=',7F12.0,/
     1'OCS=',7F12.0)
  110 FORMAT('CAPACITY INCREMENTS ARE: ',/10(4F12.0,/))
  105 format('MODES:'/,'1 = EXPRESSWAY (PEAK)'/,'2 = EXPRESSWAY(
     1OFF-PEAK)'/,'3 = SUBWAY (PEAK)'/,'4 = SUBWAY (OFF-PEAK)'//)
  100 format('A=',F7.5/,'VECTOR B=',4F8.3//,'ARRAY C=',4(4F10.4/,8X))
  101 format(/'X1=',F4.2,' X2=',F4.2/,'INITIAL GUESS VECTOR Z=',
     14F10.4/'AGGRIGATE DEMAND FUNCTION CONSTANTS: A0=',F12.4,',
     1A1=',F8.5,5X,'Z=',4F6.2/)
  103 format('P0= ',4F8.4/,'INITIAL CAPACITIES= ',4F12.4//)
      WRITE(3,555) POP
  555 FORMAT('POP=',4F7.2)
      write(3,200)
  200 format(132('-'))
C  INITIALIZE CONSTANTS FOR USE IN PROGRAM
      PVOCE=0.0
      PVOCS=0.0
      PVRS=0.0
      PVCCS=206400000.
      PVCCE=97500000.
      PVRE=0.0
      OCSUB=1000000.
      OCEI=29000.
      NPVS=-PVCCS-PVCCE
C  BEGIN COMPUTATION
C
   89 Write(3,2) year
    2 format(//'DATA FOR YEAR ',I2,':'/)
      write(6,8) year
      write(3,112)
  112 format(16x,'EX(PEAK)',4X,'EX(OFF)',5X,'SUB(PEAK)',4X,'SUB(OFF)')
C  CALCULATE EXPANSION CRITERION
      CRTRN=RR$(CCS(ns)+CCE(ne))+OCE(ne)+ocs(ns)
      CRTRN1=RR$CCE(ne)+OCE(ne)
      CRTRN2=RR$CCS(ns)+OCS(ns)
C  DERIV CALCULATES THE NECESSARY PARTIAL DERRIVATIVES AND FINDS THE
C  JACOBIAN MATRIX: DECOMP AND SOLVE THEN SOLVE THIS SYSTEM
   90 call deriv (j,w)
      call decomp(4,4,j,cond,ipvt,work)
```

```
      call solve(4,4,Jsv,ipvt)
      test=0.0
      do 1 i=1,4
      test=test+dabs(w(i))
C  UPDATE QUANTITY VECTOR TO CLOSE IN ON THE TRUE SOLUTION
   1  Q(I)=Q(I)-Y(I)
C  TEST FOR ACCURACY OF THE SOLUTION
      if(test.gt.tau) goto 90
C ONCE WE HAVE A GOOD APPROXIMATION TO THE SOLUTION WE CAN CALCULATE
C THE QUANTITIES,SURPLUSES AND REVENUES
      QQ=a0*PP**a1
      surpls=0.0
      TR=0.0
      DO 22 I=1,4
      REV(I)=REVV(I)
C  THIS PROGRAM IS ON A DAILY BASIS SO WE NEED YEARLY BASIS IE 365
  22  TR=TR+REV(I)*365.
      IF(.NOT.EXPRSS.AND..NOT.SUBWAY) TR1=TR
      do 50 i=1,4
C FIND AUD AND SURPLS
  50  Surpls=surpls+srpls(p0,(i),c(i),a(i),z(i),x1)
      aud=(1./a0)**(1./a1)*(a1/(a1+1.))*
     1(QQ**((A1+1.)/A1)-1.)
C TEST WHICH STAGE OF THE FOUR PART SOLUTION WE ARE CURRENTLY AT
C AND BRANCH ACCORDINGLY
      IF(.NOT.EXPRSS.AND..NOT.SUBWAY) GOTO 6
      IF(EXPRSS.AND.SUBWAY.AND.365.*(AUD-SURPLS)-SAVE.GE.CRTRN)
     1GOTO 13
      IF(EXPRSS.AND.SUBWAY) GOTO 14
      IF(.NOT.EXPRSS.AND.SUBWAY.AND.365.*(AUD-SURPLS)-SAVE.GE.
     1CRTRN2) GOTO 15
      IF(.NOT.EXPRSS.AND.SUBWAY) GOTO 16
      IF(EXPRSS.AND..NOT.SUBWAY.AND.365.*(AUD-SURPLS)-SAVE.GE.
     1CRTRN1) GOTO 17
      GOTO 18
C  INCREMENT FOR THE NEXT YEAR
  21  Year=year+1
      a0=a0*rate
      if(year.eq.41) goto 88
      goto 89
C CASE OF NEITHER EXPANDED YET
   6  do 7 i=1,2
      c(i+2)=c(i+2)+cinc(ns,i+2)
   7  c(i)=c(i)+CINC(ne,I)
      write(3,999) p,Q,REV,PP,aa
 999  format(//'P(i),i=1,4=',4f12.2/,'Q(i),i=1,4=',4f12.2,
     1/,'EXISTING',//'REVENUES=  ',4F12.2,//'P=',F12.2,5X,
     1'Q=',F12.2)
      save=365.*(aud-surpls)
      EXPRSS=.TRUE.
      SUBWAY=.TRUE.
      goto 90
C CASE OF EXPANSION FOR BOTH FACILITIES
  13  DO 19 I=1,2
      c0(i+2)=c0(i+2)+cinc(ns,i+2)
      C0(I)=C0(I)+CINC(ne,I)
      c(i+2)=c0(i+2)
  19  C(I)=C0(I)
      OCSUB=OCSUB+OCS(NS)
      OCEX=OCEX+OCE(NE)
      write(3,111) p,Q,REV,PP,aa
 111  format(//'P(i),i=1,4=',4f12.2/,'Q(i),i=1,4=',4f12.2,
     1/,'BOTH',//'REVENUES=  ',4F12.2,//'P=',f12.2,5x,
```

```
     1'Q=',F12.2)
      DS=365.*(AUD-SURPLS)-SAVE
      NPVS=NPVS+(DS+SAVE-CCS(ns)-CCE(ne)-OCS(ns)-OCE(ne))/
     1(1.+ir)**year
      PVRS=PVRS+(365.*(REV(3)+REV(4)))/(IR+1.)**YEAR
      PVRE=PVRE+(365.*(REV(1)+REV(2)))/(IR+1.)**YEAR
      PVCCE=PVCCE+CCE(NE)/(1.+IR)**YEAR
      PVCCS=PVCCS+CCS(NS)/(1.+IR)**YEAR
      ne=ne+1
      ns=ns+1
      WRITE(3,106) DS,CO
  106 FORMAT(/'DS=',F15.2,' (BOTH EXPANDED)'//,'BOTH EXPANDED,
     1NEW CAPACITIES=',4F7.0)
      WRITE(3,202) NPVS
      PVOCS=PVOCS+OCSUB/(1.+IR)**YEAR
      PVOCE=PVOCE+OCEX/(1.+IR)**YEAR
      CALL CHART(PVRS,PVRE,PVOCS,PVOCE,PVCCS,PVCCE
      WRITE(3,200)
      EXPRSS=.FALSE.
      SUBWAY=.FALSE.
      GOTO 21
C CASE: BOTH NOT EXPANDED, CHECK SUBWAY ALONE
   14 C(1)=CO(1)
      C(2)=CO(2)
      EXPRSS=.FALSE.
      write(3,113) P,Q,REV,PP,oo
  113 format(/'P(i),i=1,4=',4f12.2,/,'Q(i),i=1,4=',4f12.2,
     17x,'BOTH',/'REVENUES=  ',4F12.2,/'P=',f12.2,5x,
     1'Q=',F12.2)
      GOTO 90
C CASE OF SUBWAY ALONE EXPANDED
   15 CO(3)=CO(3)+CINC(ns,3)
      CO(4)=CO(4)+CINC(ns,4)
      NPVS=NPVS+(365.*(AUD-SURPLS)-CCS(ns)-OCS(ns)-OCE(ne))/
     1(1.+ir)**year
      OCSUB=OCSUB+OCS(NS)
      PVRE=PVRE+365.*(REV(1)+REV(2))/(1.+IR)**YEAR
      PVRS=PVRS+365.*(REV(3)+REV(4))/(1.+IR)**YEAR
      PVCCS=PVCCS+CCS(NS)/(1.+IR)**YEAR
      ns=ns+1
      FLAG1=.TRUE.
C CASE OF SUBWAY ALONE NOT EXPANDED
   16 DS1=365.*(AUD-SURPLS)-SAVE
      write(3,114) P,Q,REV,PP,oo
  114 format(/'P(i),i=1,4=',4f12.2,/,'Q(i),i=1,4=',4f12.2,
     17x,'SUBWAY',/'REVENUES=  ',4F12.2,/'P=',f12.2,5x,
     1'Q=',F12.2)
      EXPRSS=.TRUE.
      SUBWAY=.FALSE.
      C(1)=C(1)+CINC(ne,1)
      C(2)=C(2)+CINC(ne,2)
      it=ns
      if(flag1) it=it-1
      C(3)=C(3)-CINC(it,3)
      C(4)=C(4)-CINC(it,4)
      GOTO 90
C CASE OF EXPRESSWAY ALONE EXPANDED; HOWEVER SUBWAY DOES TAKE PRIORITY
   17 FLAG2=.NOT.FLAG1
      IF(FLAG1) GOTO 18
      CO(1)=CO(1)+CINC(ne,1)
      CO(2)=CO(2)+CINC(ne,2)
      NPVS=NPVS+(365.*(AUD-SURPLS)-OCE(ne)-CCE(ne)-OCS(ns))/
     1(1.+ir)**year
```

```
      OCEX=OCEX+OCE(NE)
      PVRS=PVRS+365.*(REV(3)+REV(4))/(1.+IR)**YEAR
      PVRE=PVRE+365.*(REV(1)+REV(2))/(1.+IR)**YEAR
      PVCCE=PVCCE+CCE(NE)/(1.+IR)**YEAR
      ne=ne+1
C CASE OF EXPRESSWAY NOT EXPANDED
   10 EXPRSS=.FALSE.
      SUBWAY=.FALSE.
      write(3,115) P,Q,REV,PP,oo
  115 format(/'P(i),i=1,4=',4f12.2/,'Q(i),i=1,4=',4f12.2,
     17x,'EXPRESSWAY',/'REVENUES=  ',4F12.2,/'P=',f12.2,5x,
     1'Q=',F12.2)
      PVOCS=PVOCS+OCSUB/(1.+IR)**YEAR
      PVOCE=PVOCE+OCEX/(1.+IR)**YEAR
      IF(FLAG1.OR.FLAG2) GOTO 25
      NPVS=NPVS+(SAVE-OCS(ns)-OCE(ne))/(1.+IR)**YEAR
      PVRS=PVRS+365.*(REV(3)+REV(4))/(1.+IR)**YEAR
      PVRE=PVRE+365.*(REV(1)+REV(2))/(1.+IR)**YEAR
   25 DS2=365.*(AUD-SURPLS)-SAVE
      DO 20 I=1,4
   20 C(I)=C0(I)
C WRITE OUT YEAR SUMMARY DATA
      WRITE(3,109) DS1,DS2
      IF(FLAG1) WRITE(3,107) C0
      IF(FLAG2) WRITE(3,108) C0
      FLAG1=.FALSE.
      FLAG2=.FALSE.
      WRITE(3,202) NPVS,PVR,PVCC,PVNR
  202 FORMAT('NPVS=',F20.2,', PVR=',F20.2,', PVCC=',F20.2,
     1', PVNR=',F20.2)
      CALL CHART(PVRS,PVRE,PVOCS,PVOCE,PVCCS,PVCCE)
      WRITE(3,200)
      GOTO 21
  107 FORMAT('SUBWAY ONLY EXPANDED, NEW CAPACITIES=',4F7.0)
  108 FORMAT('EXPRESSWAY ONLY EXPANDED, NEW CAPACITIES=',4F7.0)
  109 FORMAT('DS=',F15.2,' (EXPANDED SUBWAY)'/,'DS=',F15.2,
     1' (EXPANDED EXPRESSWAY)')
   88 stop
      end
      subroutine decomp(ndim,n,a,cond,ipvt,work)
      integer ndim,n
      double precision a(ndim,n),cond,work(n)
      integer ipvt(n)
c
c     Decomposes a real matrix by Gaussian Elimination
c     and estimates the condition of the matrix.
c
c     Use SOLVE to compute solutions to linear systems.
c
c     INPUT:
c       ndim = declared row dimension of matrix containing A.
c       n = order of the matrix.
c       A = matrix to be triangularized.
c
c     OUTPUT:
c       A contains an upper triangular matrix U and a
c         permuted version of a lower triangular matrix
c         I-L so that (permutation matrix)A=LU
c
c       cond = an estimate of the condition of A.
c         For the linear system Ax=b, changes in A and b
c         may cause changes cond-times as large in x.
c         If cond+1.0.eq.cond, A is singular to working
```

```
c        precision. Cond is set to 1.0E+32 if exact sing-
c        ularity is detected.
c
c      ipvt = the pivot vector.
c        ipvt(k) = the index of the kth pivot row.
c        ipvt(n) = (-1)**(number of interchanges).
c
c      work space... the vector work must be declared and
c        included in the call. Its input contents are ignored.
c        Its output contents are usually unimportant.
c
c     The determinant of A can be obtained on output by
c        Det(A)=ipvt(n)*A(1,1)*...*A(n,n).
c
      double precision ek,t,anorm,ynorm,znorm
      integer nm1,i,j,k,kp1,kb,kal,m

      ipvt(n)=1
      if (n.eq.1) goto 80
      nm1=n-1
c
c  compute 1-norm of A.
c
      anorm=0.0
      do 10 j=1,n
        t=0.0
        do 5 i=1,n
          t=t+dabs(a(i,j))
    5   continue
        if (t.gt.anorm) anorm=t
   10 continue
c
c  Gaussian elimination with partial pivoting.
c
      do 35 k=1,nm1
        kp1=k+1
c
c  find pivot
c
        m=k
        do 15 i=kp1,n
          if (dabs(a(i,k)).gt.dabs(a(m,k))) m=i
   15   continue
        ipvt(k)=m
        if (m.ne.k) ipvt(n)=-ipvt(n)
        t=a(m,k)
        a(m,k)=a(k,k)
        a(k,k)=t
        if (t.eq.0.0) goto 35
        do 20 i=kp1,n
          a(i,k)=-a(i,k)/t
   20   continue
c
c  interchange and eliminate by columns.
c
        do 30 j=kp1,n
          t=a(m,j)
          a(m,j)=a(k,j)
          a(k,j)=t
          if (t.eq.0.0) goto 30
          do 25 i=kp1,n
            a(i,j)=a(i,j)+a(i,k)*t
   25     continue
```

```
   30    continue
   35  continue
c
       do 50 k=1,n
         t=0.0
         if (k.eq.1) goto 45
         km1=k-1
         do 40 i=1,km1
           t=t+a(i,k)*work(i)
   40    continue
   45    ek=1.0
         if (t.lt.0.0) ek=-1.0
         if (a(k,k).eq.0.0) goto 90
         work(k)=-(ek+t)/a(k,k)
   50  continue
       do 60 kb=1,nm1
         k=n-kb
         t=0.0
         kp1=k+1
         do 55 i=kp1,n
           t=t+a(i,k)*work(k)
   55    continue
         work(k)=t
         m=ipvt(k)
         if (m.eq.k) goto 60
         t=work(m)
         work(m)=work(k)
         work(k)=t
   60  continue
c
       vnorm=0.0
       do 65 i=1,n
         vnorm= vnorm+dabs(work(i))
   65  continue
c
c  solve A*z=y
c
       call solve(ndim,n,a,work,ipvt)
c
       znorm=0.0
       do 70 i=1,n
         znorm=znorm+dabs(work(i))
   70  continue
c
c  estimate condition.
c
       cond=anorm*znorm/vnorm
       if (cond.lt.1.0) cond=1.0
       return
   80  cond=1.0
       if(a(1,1).ne.0.0) return
   90  cond=1.0e+32
       return
       end
       subroutine solve(ndim,n,a,b,ipvt)
c
       integer ndim,n,ipvt(n)
       double precision a(ndim,n),b(n)
c
c  Solution of linear system A*x=b
c  Do not use if DECOMP has detected singularity.
c
c  INPUT:
```

```
c     ndim = declared row dimension of array containing A.
c     n = order of matrix
c     A = triangularized matrix obtained from DECOMP.
c     b = right-hand-side vector.
c     ipvt = pivot vector obtained from DECOMP.
c
c  OUTPUT:
c     b = solution vector x.
c
      integer kb,km1,nm1,kp1,i,k,n
      double precision t
c
c  Forward elimination.
c
      if (n.eq.1) goto 50
      nm1=n-1
      do 20 k=1,nm1
        kp1=k+1
        m=ipvt(k)
        t=b(m)
        b(m)=b(k)
        b(k)=t
        do 10 i=kp1,n
          b(i)=b(i)+a(i,k)*t
   10   continue
   20 continue
c
c  Back substitution.
c
      do 40 kb=1,nm1
        km1=n-kb
        k=km1+1
        b(k)=b(k)/a(k,k)
        t=-b(k)
        do 30 i=1,km1
          b(i)=b(i)+a(i,k)*t
   30   continue
   40 continue
   50 b(1)=b(1)/a(1,1)
      return
      end
C*********************************************************************
C
C  THIS PROGRAM CALCULATES THE PARTIAL DERRIVATIVES AND SETS UP
C  THE JACOBIAN MATRIX FOR USE BY DECOMP
C
C*********************************************************************
      subroutine deriv(t,v)
      implicit double precision (a-h,o-z)
      dimension cc(4,4),b(4),t(4,4),v(4),s(4),a(4),dr,da(4,4)
      dimension dtrda(4),dada(4,4),p(4),c(4),p0(4),z(4),
     1POP(4)
C  SAME COMMON BLOCK USED IN ROAD.FOR
      common/save/ a,b,cc,s,r,ra,PP,p0,P0,c,a0,al,x1,x2,z
C  FIND THE SHARES
      do 1 i=1,4
      s(i)=b(i)
      do 2 j=1,4
C  THIS IS A CHECK ON ILL-CONDITIONED SYSTEMS
      IF(B(J).GE.C(J)) B(J)=C(J)-1.
      if(a(J).le.0) a(J)=10.
C FIND THE PRICE ASSOCIATED WITH THE GUESSED QUANTITY
      P(J)=POP(J)+P0(J)+Z(J)*(C(J)/(C(J)-B(J)))**1X18
```

```
     1(1.+X2*D(J)/(C(J)-D(J)))
      s(i)=s(i)+cc(i,j)*dlos(p(j))
      if(j.ne.i) goto 3
C FIND dPi by dQj (partial derivative)
      d=da(i,j)=0.0
      goto 2
 3    d=da(i,j)=(c(i)-a(i)*(x2-1.)*c(i)**x1+((x2-1)*a(i)*c(i)**
     1x1+c(i)**(x1+1.))*(x1+1.))/((c(i)-a(i))**(x1+2.))
 2    continue
 1    continue
C FIND dSi by dQj
      do 4 i=1,4
      do 5 j=1,4
      dsda(i,j)=cc(i,j)/p(j)*d=da(j,j)
 5    continue
 4    continue
C FIND d(translog fn) by dQi
      do 6 i=1,4
      dtrda(i)=b(i)/p(i)*d=da(i,i)
      do 7 j=1,4
      dtrda(i)=dtrda(i)+.5*cc(j,i)*(dlos(p(j))/p(i)*d=da(i,i)+
     1dlos(p(i))/p(i)*d=da(i,i))
 7    continue
 6    continue
C FIND THE PRICE GIVEN BY THE TRANSLOG DEMAND FUNCTION
      PP=a
      do 8 i=1,4
      PP=PP+b(i)*dlos(p(i))
      do 9 j=1,4
      PP=PP+0.5*cc(i,j)*dlos(p(i))*dlos(p(j))
 9    continue
 8    continue
      PP=dexp(PP)
      do 10 i=1,4
 10   dtrda(i)=dtrda(i)*PP
C SET UP THE RIGHT-HAND-SIDE OF THE SYSTEM FOR SOLVE
      do 11 i=1,4
      v(i)=a00s(i)*PP**(a1+1.)/a(i)->(i)
      do 12 j=1,4
      if (j.ne.i) goto 13
C SET UP THE JACOBIAN MATRIX (T HERE) FOR DECOMP
      t(i,j)=a0/a(i)*(s(i)*(a1+1.)*PP**a1*dtrda(j)+dsda(i,j)*
     1PP**(a1+1.))
      goto 12
 13   t(i,j)=(a(i)*(a00s(i)*(a1+1.)*PP**a1*dtrda(i)+a0*PP**(a1+1.)
     1*dsda(i,i))-a00s(i)*PP**(a1+1.))/(a(i)**2)-d=da(i,i)
 12   continue
 11   continue
      return
      end
C*****************************************************************
C
C  THIS ROUTINE CALCULATES THE REVENUE FOR A GIVEN MODE i.
C  FOR USE THIS PROGRAM MUST BE LINKED WITH ROAD1 THE MAIN
C  PROGRAM.
C  VARIABLE DEFINITIONS ARE AS IN THE MAIN PROGRAM.
C  THERE IS NO NEED TO REFERENCE THE VARIABLES IN THE CALL
C  STATEMENT AS ALL RELEVANT VARIABLES ARE STORED IN A COMMON
C  BLOCK.
C  THE FORMULA FOR CALCULATION OF THE REVENUE SIMPLY IS:
C             Q*(MC-AC)      :  (X2.ME.0)
C             D*P0           :  (X2.EQ.0)
C
```

```
C*****************************************************************
      DOUBLE PRECISION FUNCTION REVV(I)
      DOUBLE PRECISION A,B(4),CC(4,4),S(4),P(4),D(4),TR,PO(4)
      DOUBLE PRECISION C(4),A0,A1,X1,X2,Z(4),GT(4),POF(4)
      COMMON/SAVE/ A,B,CC,S,P,D,TR,PO,POF,C,A0,A1,X1,X2,Z
      IF(X2.EQ.0) GOTO 1
      revv=a(i)*z(i)*x1*(c(i)/(c(i)-a(i)))**(x1+1.)
      RETURN
    1 REVV=PO(I)*BD(I)
      RETURN
      END
C*****************************************************************
C
C  THIS ROUTINE ONLY CALCULATES THE SURPLUS FOR A GIVEN
C  MODE.
C  THE FORMULA IS:
C  QNAC= AREA UNDER MC CURVE.
C
C*****************************************************************
      double precision function srpls(fc,a,z,x)
      srpls=a*(f*z*(c/(c-a)))**x
      return
      end
C*****************************************************************
C
C  THIS ROUTINE PRINTS OUT A CHART SUMMARIZING THE CAPITAL AND
C  OPERATING COSTS AND REVENUE VALUES TO DATE.
C  THIS ROUTINE MUST BE LINKED TO ROAD TO BE USED.
C  CALL CHART WITH THE PARAMETERS IN THE SAME ORDER AS BELOW.
C
C*****************************************************************
      SUBROUTINE CHART(PURS,PURE,PVOCS,PVOCE,PVCCS,PVCCE,
      IMPLICIT DOUBLE PRECISION (A-Z)
      WRITE(3,100)
  100 FORMAT(/,15X,'EX-WAY',11X,'SUNWAY',12X,'TOTAL')
      WRITE(3,101) PURE,PURS,PURE+PURS
  101 FORMAT('PUR ',F16.2,'+',F16.2,'=',F18.2)
      WRITE(3,102) PVOCE,PVOCS,PVOCE+PVOCS
  102 FORMAT('PVOC',F16.2,'+',F16.2,'=',F18.2)
      WRITE(3,103) PVCCE,PVCCS,PVCCE+PVCCS
  103 FORMAT('PVCC',F16.2,'+',F16.2,'=',F18.2)
      WRITE(3,104)
  104 FORMAT(4x,52('-'))
      WRITE(3,105)PURE-PVOCE-PVCCE,PURS-PVOCS-PVCCS,
     1PURE+PURS-PVOCE-PVOCS-PVCCE-PVCCS
  105 FORMAT('PVAR',F16.2,'+',F16.2,'=',F18.2)
      RETURN
      END
```